ISSN 0027-1330, Moscow University Mechanics Bulletin, 2016, Vol. 71, No. 2, pp. 39–41. © Allerton Press, Inc., 2016.
Original Russian Text © A.G. Kalugin, 2016, published in Vestnik Moskovskogo Universiteta, Matematika. Mekhanika, 2016, Vol. 71, No. 2, pp. 57–59.

BRIEF COMMUNICATIONS

Orientational Instability of a Lyotropic Nematic Liquid Crystal Layer

A. G. Kalugin

Moscow State University, Faculty of Mechanics and Mathematics, Leninskie Gory, Moscow, 119899, Russia; e-mail: kalugin@mech.math.msu.su

Received December 10, 2014; in final form, December 22, 2015

Abstract—The problem of periodic domain initiation in a thin lyotropic nematic liquid crystal layer is studied. This layer has a planar director initial orientation, but the anchoring energy is minimized by the homeotropic one. The periodic structures whose wave vector is perpendicular to the director exist during the director reorientation process from the planar orientation to the homeotropic one when the reorientation wave front appears. It is shown that the divergent terms of the Frank orientation elasticity energy plays an important role in this effect. The saddle-splay Frank constant and the anisotropic anchoring energy coefficient are estimated.

DOI: 10.3103/S0027133016020035

1

Various periodic stripe domains are observed in nematic liquid crystals. These structures can appear spontaneously or as a result of external actions [1]. Such effects can be caused by the divergent terms of the Frank orientation elasticity energy [2]. In this paper the appearance of periodic structures is theoretically described for the case of lyotropic nematic liquid crystals (more specifically, for the case of a disulfoindanthrone–water system [3]).

Let us consider a liquid crystal layer bounded by two walls. In the initial state, the orientation unit vector $\mathbf{n}$ (also called the director) is directed along the walls. By virtue of the boundary conditions, however, the homeotropic orientation corresponds to the anchoring energy minimum when the director is perpendicular to the boundary of the medium. With the course of time, the reorientation process is observed in the layer: first, this process is observed in some segments of the cell and, then, is propagated through the entire domain in the form of a wave front. When the reorientation front is perpendicular to the initial position of the director, some periodic structures appear near the front in the unperturbed zone; the wave vector of these structures is perpendicular to the orientation vector. Finally, the periodic structures are propagated through the entire unperturbed zone [3]. It is found experimentally that the appearance time of periodic stripe domains is much less than the reorientation time over the entire cell and that their period is almost unchanged during the motion of the reorientation front. A dynamic model of this effect is also considered in [3].

In this paper we describe the appearance of such cells with consideration of the divergent terms of the Frank elasticity energy whose one-constant approximation is as follows [1, 4]:

$$2F_V = K\nabla_i n_j \nabla^i n^j + K_{24}\left(\nabla_i n_j \nabla^j n^i - (\nabla_k n^k)^2\right). \qquad (1)$$

Here K and K_{24} are the constant Frank coefficients. The second term of (1) is of divergent form; hence, this term has no effect on the equations inside the volume and is used in the boundary conditions. The Rapini–Papoular anchoring energy is given by the formula [1, 4]

$$2F_S = 2\gamma + W\left(1 - (\sin\Omega\sqrt{1-n_\nu^2} + \cos\Omega|n_\nu|)^2\right),$$

where γ and W are constant coefficients and Ω is the angle between the easy orientation axis and the outer unit normal ν to the surface; here $\Omega \in [0, \pi/2]$. This axis specifies the director's orientation in the unperturbed state and can rotate about the cone whose generatrices makes the angle Ω with the normal. The minimum of the anchoring energy is attained when the normal to the surface, the easy orientation axis, and the director belong to the same plane [5]. Then, the equilibrium equations and the boundary conditions can be written as [5–7]

$$\left(\delta_k^j - n^j n_k\right)\left(\frac{\partial F_V}{\partial n^j} - \nabla_i\left(\frac{\partial F_V}{\partial \nabla_i n^j}\right)\right) = 0\,, \tag{2}$$

$$\left(\delta_k^j - n^j n_k\right)\left(\frac{\partial F_V}{\partial \nabla_i n^j}\nu_i + \frac{dF_S}{dn_\nu}\nu_j\right) = 0\,. \tag{3}$$

The pressure can be determined from the equilibrium conditions $\nabla_i(p + F_V) = 0$ when the orientation vector field is found. The equations expressed by (2) and (3) are projected onto the plane perpendicular to $\mathbf{n}$; as a result, the Lagrange undetermined multipliers are eliminated (these multipliers arise from the condition that the director is of constant length).

2

Let us analyze the equilibrium of the nematic occupying the layer $-h \le z \le h$, where (x, y, z) is the chosen Cartesian coordinate system. In the unperturbed state, the director is parallel to the walls: $\mathbf{n} = (\cos\psi_0, \sin\psi_0, 0)$. In (3) we put $\Omega = 0$; the unperturbed planar orientation is not a minimum state of the anchoring energy. Now we consider the perturbed solution in the form of small horizontal and vertical deviations from the initial state; the corresponding deviation angles are denoted by ψ and θ. Then, the linearized equilibrium equations (2) for the perturbations can be written as

$$\Delta\theta = 0\,, \qquad \Delta\psi = 0\,. \tag{4}$$

For $z = \pm h$, the linearized boundary conditions are specified by the equations [8]

$$K\psi_z = K_{24}(\theta_y \cos\psi_0 - \theta_x \sin\psi_0)\,, \tag{5}$$

$$\mp K\theta_z \pm K_{24}(\psi_x \sin\psi_0 - \psi_y \cos\psi_0) = -W\theta\,, \tag{6}$$

where the subscripts x, y, and z indicate the partial derivatives with respect to the corresponding coordinates, the condition expressed by (5) is valid on both the boundaries, and in (6) the upper and lower signs correspond to the upper and lower boundaries, respectively. The unperturbed state corresponds to the maximum of the anchoring energy; hence, Eq. (6) differs from the boundary conditions discussed in [8] by the opposite sign in the right-hand side.

The solutions to Eqs. (4) are sought in the form

$$\psi = (C_1 \exp(kz) + C_2 \exp(-kz)) \cos kx + (C_3 \exp(kz) + C_4 \exp(-kz)) \sin kx,$$

$$\theta = (D_1 \exp(kz) + D_2 \exp(-kz)) \cos kx + (D_3 \exp(kz) + D_4 \exp(-kz)) \sin kx,$$

where $k > 0$ is a real number and C_i and D_i are arbitrary constants. The boundary conditions (5) and (6) are used to determine the amplitudes of perturbations. As a result, we obtain a system of homogeneous linear equations for C_i and D_i; this system consists of two subsystems with equal determinants. A nontrivial solution can be found by equating these determinants to zero, which leads to the equations

$$k(K^2 - K_{24}^2 \sin^2\psi_0) = WK\,\mathrm{th}(kh)\,, \qquad k(K^2 - K_{24}^2 \sin^2\psi_0) = WK\,\mathrm{cth}(kh).$$

From these equations we conclude that, for $k > 0$ and $\psi_0 = \pi/2$, such a nontrivial solution exists if $|K_{24}| < K$. This conclusion is in contradiction with the experimental estimates for K_{24} given in [9, 10]. For a lyotropic nematic liquid crystal used in the experiments of [3], however, the Frank coefficients differ from one another by an order of magnitude compared to the Frank coefficients for typical thermotropic liquid crystals [11, 12]; hence, the above inequality can be valid.

From the above-mentioned experiments and taking into account that the observed period of perturbations is almost coincident with the thickness of the layer, we conclude that the following upper estimate is valid for the anisotropic component of the anchoring energy: $W/K < 10^6$ 1/m.

Thus, the divergent terms of the Frank orientation elasticity energy are the cause of periodic structures in a lyotropic nematic liquid crystal. Our approach allows one to experimentally estimate a value of the saddle-splay elastic constant K_{24} for such a medium.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project no. 14–01–00361).

REFERENCES

1. S. A. Pikin, *Structural Transformations in Liquid Crystals* (Nauka, Moscow, 1981; Gordon and Breach, New York, 1991).
2. G. Barbero, L. R. Evangelista, and I. Lelidis, "Spontaneous Periodic Distortions in Nematic Liquid Crystals: Dependence on the Tilt Angle," Phys. Rev. E **67**, 051708-1–4 (2003).
3. A. V. Golovanov, A. V. Kaznacheev, and A. S. Sonin, "Orientational Instability of a Flowing Lyotropic Nematic," Izv. Akad. Nauk, Ser. Fiz. **62** (8), 1658–1661 (1998) [Bull. Russ. Acad. Sci. Phys. **62** (8), 1338–1340 (1998)].
4. A. N. Golubiatnikov and A. G. Kalugin, "On Short Surface Waves in Anisotropic Fluids," Vestn. Mosk. Univ., Ser. 1: Mat. Mekh., No. 1, 42–43 (2001)
5. A. G. Kalugin and A. N. Golubyatnikov, "Equilibrium Shape of a Drop of a Nematic Liquid-Crystal Droplet," Tr. Mat. Inst. im. V.A. Steklova, Akad. Nauk. SSSR **223**, 171–177 (1998). [Proc. Steklov Inst. Math. **223**, 168–174 (1998)].
6. M. Igosheva and A. Kalugin, "Capillary Waves in Nematic Liquid Crystal," Mol. Cryst. Liq. Cryst. **526** (1), 10–17 (2010).
7. A. G. Kalugin, "The Role of Divergent Terms in the Frank Energy of Nematic Liquid Crystals," Vestn. Mosk. Univ., Ser. 1: Mat. Mekh., No. 1, 69–71 (2013) [Moscow Univ. Mech. Bull. **68** (1), 32–34 (2013)].
8. A. G. Kalugin, "On the Equilibrium of the Layer of a Nematic Liquid Crystal with an Inhomogeneous Boundary," Izv. Akad. Nauk, Mekh. Zhidk. Gaza, No. 2, 3–7 (2015) [Fluid Dyn. **50** (2), 181–185 (2015)].
9. R.D. Polak, G. P. Crawford, B. C. Kostival, et al., "Optical Determination of the Saddle-Splay Elastic Constant K_{24} in Nematic Liquid Crystals," Phys. Rev. E **49** (2), 978–981 (1994).
10. A. Sparavigna, O. D. Lavrentovich, and A. Strigazzi, "Periodic Stripe Domains and Hybrid-Alignment Regime in Nematic Liquid Crystals: Threshold Analysis," Phys. Rev. E **49** (2), 1344–1352 (1994).
11. A. V. Golovanov, A. V. Kaznacheev, and A. S. Sonin, "Concentration Dependencies of Viscoelastic Properties of Chromonic Nematics," Izv. Akad. Nauk, Ser. Fiz. **60** (4), 43–46 (1996) [Bull. Russ. Acad. Sci. Phys. **60** (4), 538–540 (1996)].
12. A. V. Golovanov, A. V. Kaznacheev, and A. S. Sonin, "Temperature Dependence of Nematic Viscoelastic Parameters in the Disulfoindanthrone–Water System," Izv. Akad. Nauk, Ser. Fiz. **59** (3), 62–67 (1995) [Bull. Russ. Acad. Sci. Phys. **59** (3), 408–412 (1995)].

Translated by O. Arushanyan