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JEAN-BAPTISTE CHARLES JOSEPH BÉLANGER (1790-1874), THE BACKWATER EQUATION AND THE BÉLANGER EQUATION

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Jean-Baptiste BÉLANGER (1790-1874)

(Courtesy of the Bibliothèque de l'Ecole Nationale Supérieure des Ponts et Chaussées)

Abstract

In an open channel, the transition from a high-velocity open channel flow to a fluvial motion is a flow singularity called a hydraulic jump. The application of the momentum principle to the hydraulic jump is commonly called the Bélanger equation, but few know that his original treatise was focused on the study of gradually varied open channel flows (BÉLANGER 1828).. The originality of BÉLANGER's (1828) essay was the successful development of the backwater equation for steady, one-dimensional gradually-varied flows in an open channel, together with the introduction of the step method, distance calculated from depth, and the concept of critical flow conditions. In 1828, Jean-Baptiste BÉLANGER understood the rapidly-varied nature of the jump flow, but he applied incorrectly the Bernoulli principle to the hydraulic jump. The correct application of momentum considerations to the hydraulic jump flow was derived 10 years later and first published by BÉLANGER (1841) Altogether Jean-Baptiste BÉLANGER's (1828,1841,1849) contributions to modern open channel hydraulics were remarkable and influenced the works by J.A.C. BRESSE, H.P.G. DARCY, A.J.C. BARRÉ de SAINT VENANT, and J.V. BOUSSINESQ.

Keywords: Jean-Baptiste BÉLANGER, Backwater equation, Gradually-varied flows, Critical flow conditions, Direct step method, Bélanger equation, Hydraulic jumps, Momentum equation, Energy equation, Open channel flows, Hydraulic engineering.

Résumé

En hydraulique des écoulements à surface libre, le nom de Jean-Baptiste BÉLANGER est souvent associé au ressaut hydraulique, et à l'application de l'équation de conservation de la quantité de mouvement, appelée l'équation de Bélanger. Une étude de son essai BÉLANGER (1828) montre que le thème principal était le calcul des écoulements à surface libre graduellement variés, avec le développement de l'équation du remous. Dans ce travail, Jean-Baptiste BÉLANGER dériva l'équation du remous avec une lucidité remarquable, et sa dérivation donne des résultats d'une précision étonnante. En discutant les singularités de l'équation du remous, il introduisit aussi les conditions d'écoulement critique, et le concept de profondeur critique, bien avant ses contemporains. Par contre, en 1828, BÉLANGER appliqua incorrectement le principe de conservation d'énergie au ressaut hydraulique; le résultat pour le cas d'un canal rectangulaire et horizontal était fondamentalement impropre, et il fut corrigé 10 ans plus tard (BÉLANGER 1841). Quoiqu'il en soit, la contribution de Jean-Baptiste BÉLANGER à l'hydraulique des écoulements à surface libre était exceptionnelle, en avance sur temps, et elle précéda les travaux de J.A.C. BRESSE, H.P.G. DARCY, A.J.C. BARRÉ de SAINT VENANT, and J.V. BOUSSINESQ.

Mots-clef: Jean Baptiste BÉLANGER, Equation du remous, Ressaut hydraulique, Conditions d'écoulement critique, Equation de conservation de la quantité de mouvement, Ecoulements à surface libre.

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List of symbols

The following symbols are used in this report :

A	flow cross-section area (m^2);
a	coefficient of the Prony resistance formula;
B	channel width (m);
b	coefficient of the Prony resistance formula;
D_H	hydraulic diameter : $D_H = 4 A/P_w$;
d	water depth (m) measured normal to the invert;
d_c	critical flow depth (m) : $d_c = \sqrt{q^2 / g}$ in a rectangular, horizontal channel;
E	specific energy (m); for a horizontal channel with hydrostatic pressure distribution: $E = d + \frac{V^2}{2 g}$;
Fr	Froude number defined as : $Fr = V / \sqrt{g d}$;
f	Darcy-Weisbach friction factor;
g	gravity constant (m/s^2);
H	total head (m);
k_s	equivalent sand roughness height (m);
L	length (m);
P_w	wetted perimeter (m);
Q	volume flow rate (m^3/s);
Re	Reynolds number: $Re = \rho V d/\mu$;
q	volume flow rate per unit width (m^2/s): $q = Q/B$;
S_f	friction slope;
S_o	bed slope : $S_o = \sin\theta$;
V	flow velocity (m/s) positive downstream;
x	longitudinal flow direction (m);
z_o	bed elevation (m) positive upwards;

Greek symbols

α	kinetic energy correction coefficient, also called Coriolis coefficient;
α'	velocity correction coefficient;
μ	dynamic viscosity of water (Pa s);
θ	bed slope angle with the horizontal, positive downwards;
ρ	water density (kg/m^3);
$\varnothing$	diameter (m);

Subscript

c	critical flow conditions;
1	upstream flow conditions;

2 downstream flow conditions;

Abbreviations

D/S downstream;

U/S upstream;

Notation

$\frac{\partial}{\partial x}$ partial differentiation with respect to x.

Glossary

BARRÉ de SAINT-VENANT: Adhémar Jean Claude BARRÉ de SAINT-VENANT (1797-1886), French engineer of the 'Corps des Ponts-et-Chaussées', developed the equation of motion of a fluid particle in terms of the shear and normal forces exerted on it (BARRÉ de SAINT-VENANT 1871).

Bélanger equation: momentum equation applied across a hydraulic jump in a horizontal channel; the equation was first derived by BÉLANGER (1841) and named after him.

BERNOULLI: Daniel BERNOULLI (1700-1782) was a Swiss mathematician, physicist and botanist who developed the Bernoulli equation in his "Hydrodynamica, de viribus et motibus fluidorum" textbook (1st draft in 1733, 1st publication in 1738, Strasbourg).

BIDONE: Giorgio BIDONE (1781-1839) was an Italian hydraulician. His experimental investigations on the hydraulic jump were published between 1819 and 1826.

BORDA: Jean-Charles de BORDA (1733-1799) was a French mathematician and military engineer. He achieved the rank of Capitaine de Vaisseau and participated to the U.S. War of Independence with the French Navy. He investigated the flow through orifices and developed the Borda mouthpiece.

BOSSUT: Abbé Charles BOSSUT (1730-1804) was a French ecclesiastic and experimental hydraulician, author of a hydrodynamic treaty (BOSSUT 1772).

BOUSSINESQ: Joseph Valentin BOUSSINESQ (1842-1929) was a French hydrodynamicist and Professor at the Sorbonne University (Paris). His treatise "Essai sur la théorie des eaux courantes" (1877) remains an outstanding contribution in hydraulic engineering literature.

Boussinesq coefficient: momentum correction coefficient named after J.V. BOUSSINESQ who first proposed it.

BRESSE: Jacques Antoine Charles BRESSE (1822-1883) was a French applied mathematician and hydraulician. He was Professor at the Ecole Nationale Supérieure des Ponts et Chaussées, Paris as the successor of J.B. BELANGER. His contribution to gradually-varied flows in open channel hydraulics is considerable (BRESSE 1860).

BUAT: Comte Pierre Louis George du BUAT (1734-1809) was a French military engineer and hydraulician. He was a friend of Abbé C. BOSSUT. Du BUAT is considered as the pioneer of experimental hydraulics. His textbook (BUAT 1779) was a major contribution to flow resistance in pipes, open channel hydraulics and sediment transport.

CARNOT: Lazare N.M. CARNOT (1753-1823) was a French military engineer, mathematician, general and statesman who played a key-role during the French Revolution.

CAUCHY: Augustin Louis de CAUCHY (1789-1857) was a French engineer from the 'Corps des Ponts-et-Chaussées'. He devoted himself later to mathematics and he taught at Ecole Polytechnique, Paris, and at the Collège de France. He worked with Pierre-Simon LAPLACE and J. Louis LAGRANGE. In fluid mechanics, he contributed greatly to the analysis of wave motion.

CHEZY: Antoine CHEZY (1717-1798) (or Antoine de CHEZY) was a French engineer and member of the French 'Corps des Ponts-et-Chaussées'. He designed canals for the water supply of the city of Paris. In 1768 he proposed a resistance formula for open channel flows called the Chézy equation. In 1798, he became Director of the Ecole Nationale Supérieure des Ponts et Chaussées after teaching there for many years.

CORIOLIS: Gustave Gaspard CORIOLIS (1792-1843) was a French mathematician and engineer of the 'Corps des Ponts-et-Chaussées' who first described the Coriolis force (i.e. effect of motion on a rotating body).

Coriolis coefficient: kinetic energy correction coefficient named after G.G. CORIOLIS who introduced first the velocity correction coefficient.

DARCY: Henri Philibert Gaspard DARCY (1805-1858) was a French civil engineer. He studied at Ecole Polytechnique between 1821 and 1823, and later at the Ecole Nationale Supérieure des Ponts et Chaussées (BROWN 2002). He performed numerous experiments of flow resistance in pipes (DARCY 1858) and in open channels (DARCY and BAZIN 1865), and of seepage flow in porous media (DARCY 1856a,b). He gave his name to the Darcy-Weisbach friction factor and to the Darcy law in porous media.

DUPUIT: Arsène Jules Etienne Juvénal DUPUIT (1804-1866) was a French engineer and economist. His expertise included road construction, economics, statics and hydraulics.

Ecole Nationale Supérieure des Ponts et Chaussées, Paris: French civil engineering school founded in 1747. The direct translation is : 'National School of Bridge and Road Engineering'. Among the directors there were the famous hydraulicians A. CHEZY and G. de PRONY. Other famous professors included B.F. de BELIDOR, J.B.C.J. BELANGER, J.A.C. BRESSE, G.G. CORIOLIS and L.M.H. NAVIER.

Ecole Polytechnique, Paris: Leading French engineering school founded in 1794 during the French Révolution under the leadership of Lazare CARNOT and Gaspard MONGE. It absorbed the state artillery school in 1802 and was transformed into a military school by Napoléon BONAPARTE in 1804. Famous professors included Augustin Louis CAUCHY, Jean Baptiste Joseph FOURIER, Siméon-Denis POISSON, Jacques Charles François STURM, among others.

EYTELWEIN: Johann EYTELWEIN (1764-1848) was a German mathematician and engineer.

Fawer jump: undular hydraulic jump.

Hydraulic jump: stationary transition from a rapid, high-velocity flow to a slower fluvial flow motion.

LAGRANGE: Joseph-Louis LAGRANGE (1736-1813) was a French mathematician (CHANSON 2007b). During the 1789 Revolution, he worked on the committee to reform the metric system. He was Professor of mathematics at the École Polytechnique from the start.

Left bank: looking downstream, towards the river mouth, the left bank is on the left.

MONGE: Gaspard MONGE (1746-1818), Comte de Péluse, was a French mathematician who invented descriptive geometry and pioneered the development of analytical geometry. He was a prominent figure during the French Revolution, helping to establish the Système métrique and the École Polytechnique, and being Minister for the Navy and colonies between 1792 and 1793.

PITOT: Henri PITOT (1695-1771) was a French mathematician, astronomer and hydraulician. He was a member of the French Académie des Sciences from 1724. He invented the Pitot tube to measure flow velocity in the Seine river (first presentation in 1732 at the Académie des Sciences de Paris).

POISSON: Siméon Denis POISSON (1781-1840) was a French mathematician and scientist. He developed the theory of elasticity, a theory of electricity and a theory of magnetism.

PRONY: Gaspard Clair François Marie Riche de PRONY (1755-1839) was a French mathematician and engineer. He succeeded A. CHEZY as director general of the Ecole Nationale Supérieure des Ponts et Chaussées, Paris during the French Revolution.

Rapidly varied flow: open channel flow characterised by large changes over a short distance (e.g. sharp-crested weir, sluice gate, hydraulic jump).

REECH: Ferdinand REECH (1805-1880) was a French naval instructor who proposed first the Reech-Froude number in 1852 for the testing of model ships and propellers.

Right bank: looking downstream, towards the river mouth, the right bank is on the right.

Roller: in hydraulic engineering, a series of large-scale turbulent eddies : e.g., the roller of a hydraulic jump.

Shock waves: in high-velocity, supercritical flows, a flow disturbance (e.g. change of direction, contraction) induces the development of shock waves propagating at the free-surface across the channel. Shock waves are called also lateral shock waves, oblique hydraulic jumps, Mach waves, cross-waves, diagonal jumps.

Stilling basin: hydraulic structure for dissipating the energy of the flow downstream of a spillway, outlet work, chute or canal structure. In many cases, a hydraulic jump is used as the energy dissipator within the stilling basin.

Supercritical flow: open channel flow characterised by a Froude number greater than unity.

Undular hydraulic jump: stationary hydraulic jump characterised by steady free-surface undulations downstream of the jump and by the absence of a formed roller. An undular jump flow is called a Fawer jump in homage to C. FAWER's (1937) work.

Weak jump: A weak hydraulic jump is characterised by a marked roller, no free-surface undulation and low energy loss. It is usually observed after the disappearance of undular hydraulic jump with increasing upstream Froude numbers.

1. Introduction

The hydraulic jump is the rapid and sudden transition from a high-velocity supercritical open channel flow to a subcritical flow (Fig. 1). Hydraulic jumps are commonly experienced in rivers and canals, in industrial applications and in manufacturing processes. A hydraulic jump is a flow singularity and discontinuity. For a horizontal rectangular channel and neglecting boundary friction, the continuity and momentum principles give a series of dimensionless relationships between the upstream and downstream flow properties:

$$\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1 + 8\,Fr_1^{\,2}} - 1\right) \tag{1}$$

$$\frac{Fr_2}{Fr_1} = \frac{2^{3/2}}{\left(\sqrt{1 + 8\,Fr_1^{\,2}} - 1\right)^{3/2}} \tag{2}$$

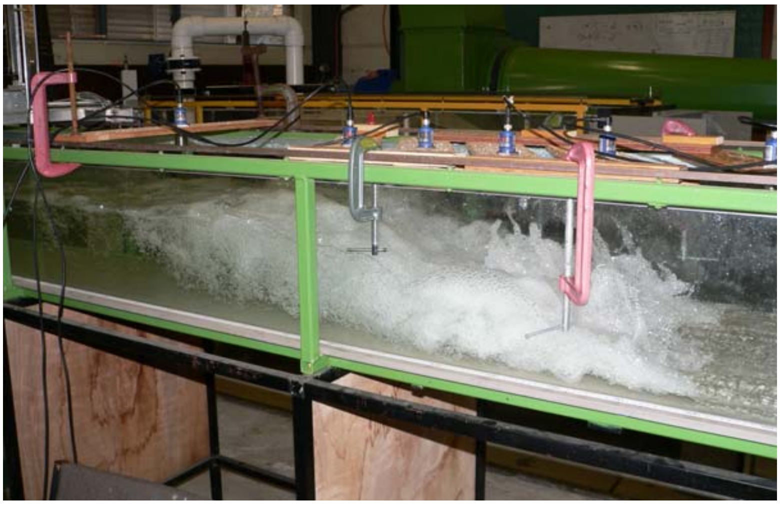
where the subscripts 1 and 2 refer to the upstream and downstream flow conditions respectively, Fr is the Froude number: $Fr = V / \sqrt{g\,d}$, d and V are the flow depth and velocity respectively, and g is the gravity acceleration. A hydraulic jump is typically classified in terms of its inflow Froude number $Fr_1 = V_1 / \sqrt{g\,d_1}$ that is always greater than unity (BÉLANGER 1828, HENDERSON 1966, CHANSON 2004). For a Froude number slightly above unity, the hydraulic jump is characterised by a smooth rise of the free-surface followed by a train of stationary free-surface undulations (Fig. 1A). For larger Froude numbers, the jump has a marked roller with large scale vortices, and the flow is characterised by significant kinetic energy dissipation and air bubble entrainment (Fig. 1B).

Historical contributions on the hydraulic jumps included the physical experiments of BIDONE (1819) performed in France in 1818, the theoretical analyses of BÉLANGER (1828,1841), the experiments of DARCY and BAZIN (1865), the solutions of BOUSSINESQ (1877) and the work of BAKHMETEFF (1932). Recent reviews encompassed HAGER (1992) and CHANSON (2007a,2009).

Jean-Baptiste BÉLANGER (Fig. 2) is commonly linked to the application of the momentum principle to the hydraulic jump: i.e., the Bélanger equation. But few people appreciate that his original paper was focused on the study of gradually varied open channel flows (BÉLANGER 1828), while his considerable influence on his contemporaries is sometimes lost. For example, his name is written on the border of one of the four facades of the Eiffel Tower together with that of the famous hydraulic engineers Jean Charles BORDA, Gaspard de PRONY, Jean-Victor PONCELET, and Jacques Antoine Charles BRESSE (Fig. 3).

The contribution of Jean-Baptiste BÉLANGER to open channel flows is re-considered herein. It is highlighted that his development of the backwater equation was remarkable for a period when numerical integration calculations were performed by hand (BÉLANGER 1828). Jean-Baptiste BÉLANGER introduced the notion of critical flow conditions as a singularity of the backwater calculations, and showed that the backwater equation cannot be solved across a hydraulic jump. He understood the rapidly-varied nature of the jump flow and the concept of supercritical inflow. Although his initial treatment of the hydraulic jump was erroneous, a later development gave the hydraulic jump equation (BÉLANGER 1841).

(A) Undular hydraulic jump: $Fr_1 = 1.1$, $d_1 = 0.104$ m, $Re = 1.1\ 10^5$, B = 0.5 m- Flow from right to left



(B) Hydraulic jump with roller: $Fr_1 = 7.9$, $d_1 = 0.018$ m, $Re = 5.9\ 10^4$, B = 0.5 m- Flow from right to left

Fig. 1 - Photographs of hydraulic jumps in the Gordon McKay Hydraulics Laboratory at the University of Queensland

Fig. 2 - Photograph of Jean-Baptiste BÉLANGER (Courtesy of the Bibliothèque de l'Ecole Nationale Supérieure des Ponts et Chaussées)

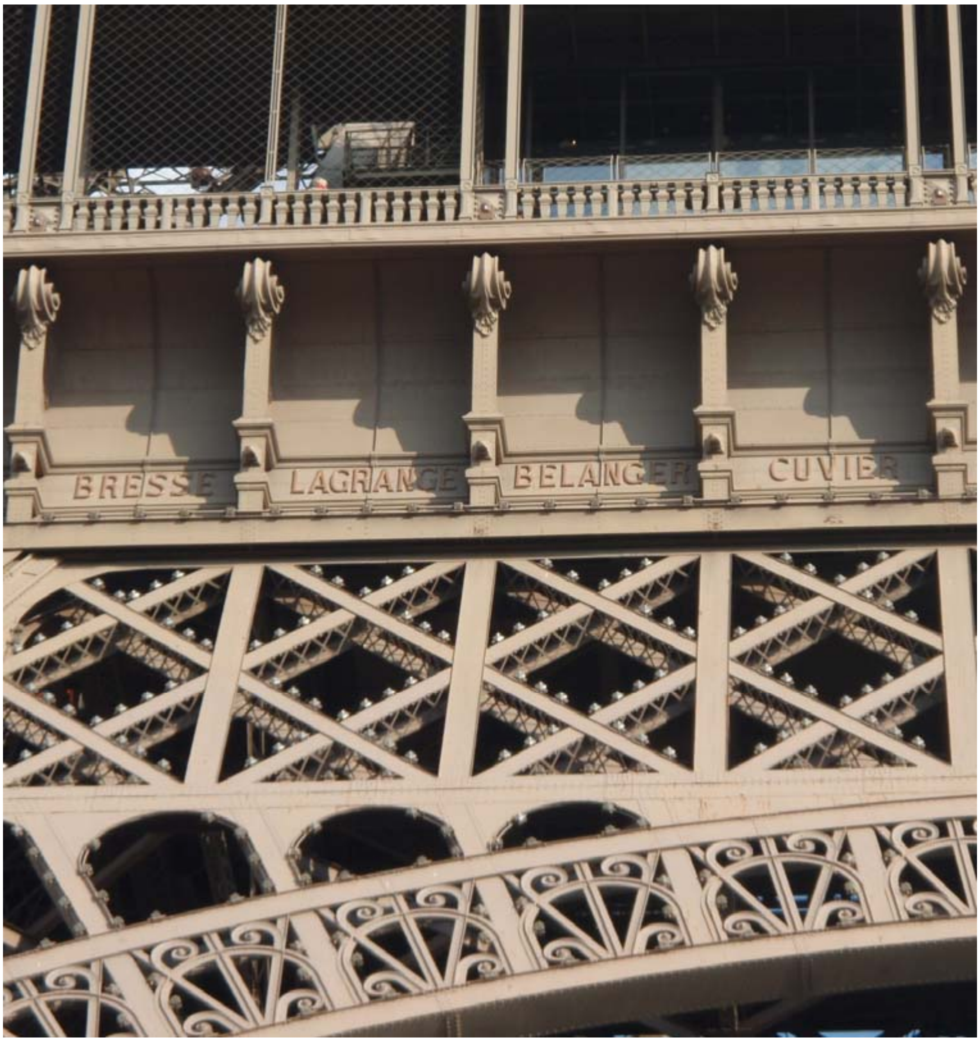


Fig. 3 - Inscription BÉLANGER on the Eiffel Tower (Tour Eiffel) between LAGRANGE and CUVIER, with BRESSE on the left - Photograph taken on 25 July 2008

2. Life of Jean-Baptiste BÉLANGER (1790-1874)

Born in Valenciennes, in northern France, on 4 April 1790 (Ref.: Birth Certificate, Parish of St Vaast en Ville, App. A), Jean-Baptiste Charles Joseph BÉLANGER was the son of Charles Antoine Aimé Joseph BÉLANGER, master locksmith, and of Jeanne Françoise Joseph FAUCONNIER. He studied in Paris at the Ecole Polytechnique ([1]), finishing second, and later at the Ecole des Ponts et Chaussées.

As Ingénieur du Corps des Ponts et Chaussées (Bridges and Roads Corps of Engineers), he started his engineering career in 1816 at La Réole. From 1821, he moved to work on the Somme navigation canal and after 1826 on the Ardennes navigation canal (La Houille Blanche 1960). It was during these two missions that he studied specifically the hydraulics of gradually-varied open channel flows. He later became a lecturer at the Ecole Centrale des Arts et Manufactures between 1838 and 1864 (Fig. 4), at the Ecole des Ponts et

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[1] in the 1808 cohort (*promotion 1808*) together with Gustave Gaspard CORIOLIS (1792-1843) (Journal de l'Ecole

Chaussées from 1841 to 1855, and at the Ecole Polytechnique from 1851 to 1860 (CHATZIS 1995). At the Ecole Centrale, one of his students was Gustave EIFFEL (1832-1923) who built the Eiffel tower and engraved his name around the first floor together with the names of 71 other scientists (Fig. 3, App. B).

Jean-Baptiste BÉLANGER retired in 1864 (HAGER 2003). He died on 8 May 1874 at Neuilly-sur-Seine, and his tomb is today in the old cemetery of Neuilly-sur-Seine (*cimetière ancien, 5ème division*).

Fig. 4 - Jean-Baptiste BÉLANGER among his academic peers at the Ecole Centrale des Arts et Manufactures (Courtesy of the Bibliothèque de l'Ecole Centrale de Paris) - BÉLANGER stands in the middle row, fifth from the right (with white hairs)

3. The analysis of the hydraulic jump: the "Bélanger equation"

From 1821, Jean-Baptiste BÉLANGER worked as a practicing engineer on a solution of gradually-varied open channel flows. He published a preliminary report in 1823 ([2]) but he felt that the work lacked theoretical foundations: "*il a senti de lui-même le désir de l'améliorer*" ('he felt himself the need to improve it'). He developed new ideas in 1826 and completed his report in 1827 (BÉLANGER 1849, p. 90). His revised document was successfully examined by the Commission des Ponts et Chaussées et des Mines on 21 July

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Polytechnique 1931).

[2] in the "Journal des Mines".

1827 ([3]) and published in 1828 (BÉLANGER 1828) (Fig. 5). The reader will find the correspondence between the original notations of BÉLANGER and modern hydraulic engineering notations in Table 1.

BÉLANGER (1828, pp. 31-36) considered the hydraulic jump as a rapidly-varied flow, across which the gradually-varied flow equation could not be applied. Based upon the experimental observations of BIDONE (1819), he treated the flow singularity (Fig. 6) by applying the energy principle using a formulation derived from a "Traité Spécial" published in 1819 by Gustave Gaspard CORIOLIS (1792-1843) ([4]): "*je me sers du théorème de Mécanique connu sous le nom d'équation des forces vives*" ('I use the Mechanics theorem known as the equation of conservation of energy').

Jean-Baptiste BÉLANGER considered the general case of a hydraulic jump in a sloping channel of irregular section. For the particular case of a flat, rectangular, prismatic channel (Fig. 6), he derived the energy equation:

$$d_2 - d_1 = \frac{V_1^2}{2\,g}\left(1 - \frac{d_1^2}{d_2^2}\right) \qquad (3)$$

Equation (3) corresponds to BÉLANGER's equation [59] (BÉLANGER 1828, p. 35).

BÉLANGER's derivation is nothing more than the solution of the energy equation in terms of the specific energy for a rectangular horizontal channel (Eq. (3)). It would give a reasonable approximation to the hydraulic jump solution for undular and weak jumps since there is very little energy loss in the jump for Froude numbers slightly greater than unity (MONTES 1986,1998), but the development is basically incorrect. Equation (3) may be rewritten in a dimensionless form as:

$$\frac{d_2}{d_1} = 1 + \frac{1}{2}\,Fr_1^2\left(1 - \left(\frac{d_2}{d_1}\right)^{-2}\right) \qquad (4)$$

This result, compared to Equation (1), is obviously wrong as illustrated in Figure 7 because it neglected the dissipation of kinetic energy. While BÉLANGER's results matched well the experimental observations for BIDONE (1819) for low Froude numbers, Equation (4) diverges from the theoretical solution (Eq. (1)) and experimental observations at larger inflow Froude numbers because the rate of energy dissipation was ignored (Fig. 7). Figure 7 presents a comparison between Equations (1) and (4), and physical measurements. The latters include the data of BIDONE (1819) used by BÉLANGER to check his results as well as new experimental observations in a 0.5 m wide rectangular channel at the University of Queensland shown in Figure 1B. Simply BÉLANGER (1828) applied incorrectly the Bernoulli principle to the hydraulic jump.

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[3] The examination report stated : "*la commission est d'avis que le travail de M. Bélanger est fait avec beaucoup de talent, et qu'il peut être fort utile; en conséquence, elle pense qu'il doit mériter à son auteur des témoignages de satisfaction et d'encouragement*" ('the committee advises that the study of M. Bélanger is talentuous and that it is useful; therefore it believes that his author deserves congratulations').

[4] Gustave Gaspard CORIOLIS studied at the Ecole Polytechnique with Jean-Baptise BÉLANGER, and he was another Ingénieur du Corps des Ponts et Chaussées. He introduced the kinetic energy correction coefficient (CORIOLIS 1836) and he is well known for his works on rotating bodies.

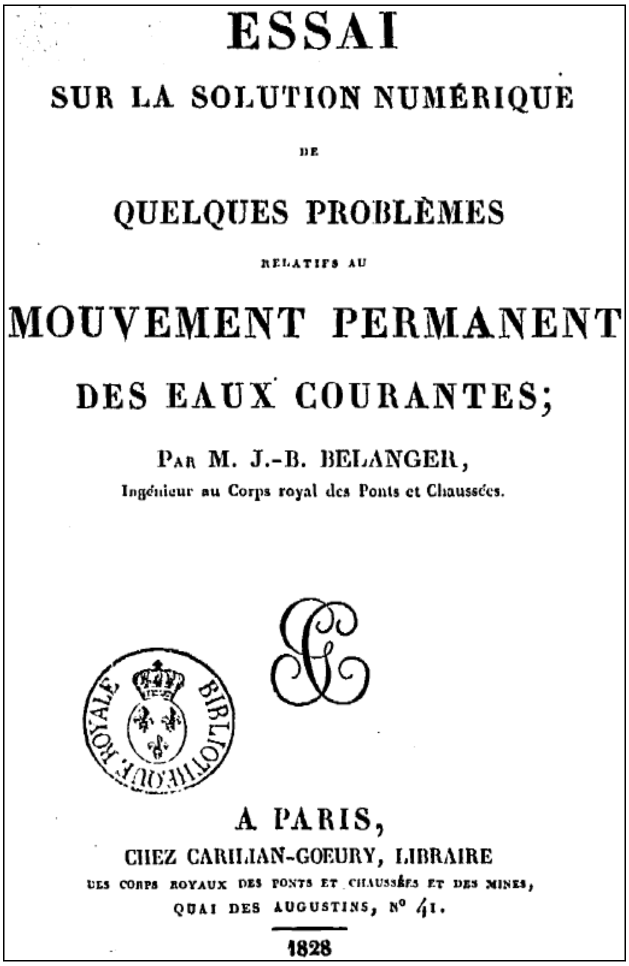
Jean-Baptiste BÉLANGER found his error in 1838: "*de nouvelles réflexions m'ont conduit en 1838 à reconnaitre que cette hypothèse n'était pas admissible*" ('new thoughts led me in 1838 to acknowledge that the assumption was incorrect') (BÉLANGER 1849, p. 91). BÉLANGER (1841) solved the momentum equation for a hydraulic jump in a flat channel. For a rectangular channel and neglecting the friction force, he obtained :

$$\frac{d_2}{d_1} = -\frac{1}{2} + \sqrt{\frac{1}{4} + 2\,\alpha'\,{Fr_1}^2} \qquad (5a)$$

where α' is a velocity correction coefficient ([5]). This reasoning became commonly accepted thereafter (BÉLANGER 1849, BRESSE 1860). For example, BRESSE (1860, p. 251) presented the same result in the form:

$$\frac{d_2}{d_1} = -\frac{1}{2} + \sqrt{\frac{1}{4} + 2\,{Fr_1}^2} \qquad (5b)$$

Equations (5a) and (5b) are mere rewritings of Equation (1).


ESSAI
SUR LA SOLUTION NUMÉRIQUE
DE
QUELQUES PROBLÈMES
RELATIFS AU
MOUVEMENT PERMANENT
DES EAUX COURANTES;
PAR M. J.-B. BELANGER,
Ingénieur au Corps royal des Ponts et Chaussées.

A PARIS,
CHEZ CARILIAN-GOEURY, LIBRAIRE
DES CORPS ROYAUX DES PONTS ET CHAUSSÉES ET DES MINES,
QUAI DES AUGUSTINS, N° 41.
1828

Fig. 5 - Cover page of the manuscript BÉLANGER (1828)

[5] Based upon BÉLANGER's (1841, pp. 88-89; 1849, pp. 82-86) development, α' should be the momentum correction coefficient, or Boussinesq coefficient, but BÉLANGER (1841,1849) gave a definition corresponding to the Coriolis

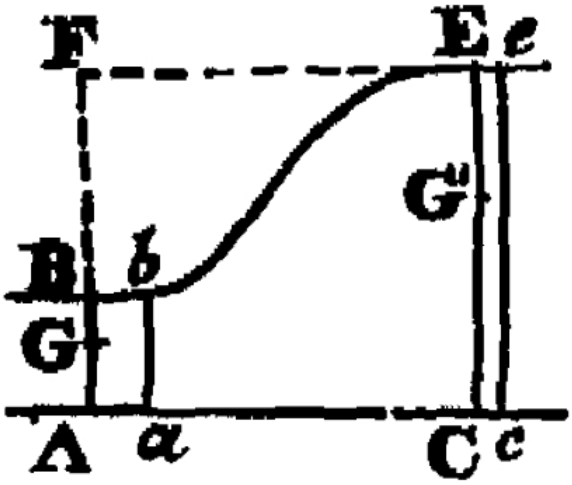


Fig. 6 - BÉLANGER's (1828) original sketch of a hydraulic jump: "coupe longitudinale du courant aux environs du ressaut"

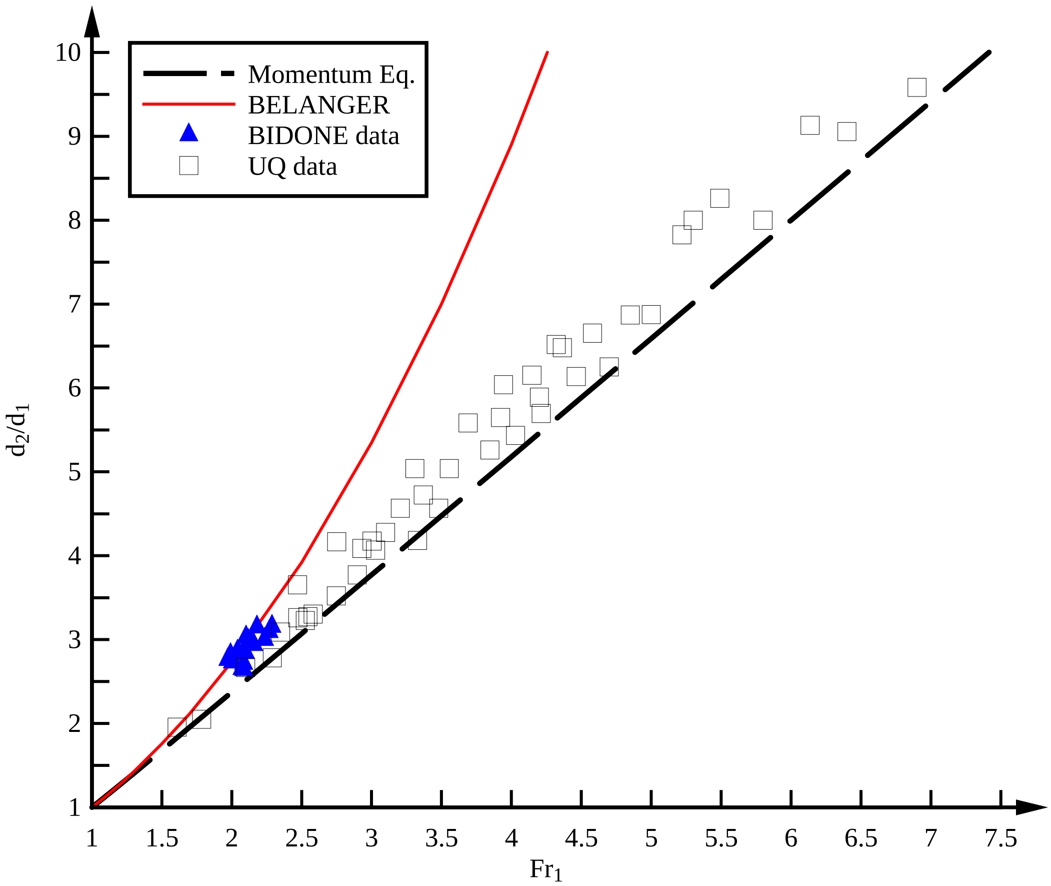


Fig. 7 - Ratio of conjugate depths for a hydraulic jump in a rectangular, horizontal, prismatic channel - Comparison between Equation (1) (black dashed lie), Equation (4) by BÉLANGER (1828) (red solid line), experimental data by BIDONE (1819) and data in a 0.5 m wide channel at the University of Queensland

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coefficient.

Table 1 - Notations used by Jean-Baptiste BÉLANGER

Definition	BÉLANGER (1828)	BÉLANGER (1841,1849)	Modern notation (+)
Water depth	h	h	d
Flow velocity	v	U	V
Discharge	--	Q	Q
Wetted perimeter	χ	χ	P_w
Cross-section area	ω	Ω	A
Angle between the invert and the horizontal	γ	--	θ
Pressure	p	--	P
Longitudinal distance	s	s	x
Distance normal to the invert	y	--	y
Depth below the free-surface	--	x	d - y
Invert slope	i	i	$S_o = \sin\theta$
Total head line slope (energy slope)	--	I	S_f
Normal depth	H	H	d_o
Rectangular channel width	λ	2 L	B
Total head above weir crest	--	Z	H

Note: (+) CHANSON (1999,2004).

Discussion

Despite his erroneous treatment, BÉLANGER (1828) demonstrated some seminal features of the hydraulic jump. He highlighted the significance of the inflow Froude number $Fr_1 = V_1 / \sqrt{g\, d_1}$, showing that a hydraulic jump occurs only for $Fr_1 > 1$: "*selon qu'on aura* $h < v^2 / g$ *ou* $h > v^2 / g$, *la formule [du ressaut] sera applicable ou ne le sera pas*" ('depending whether $h < v^2 / g$ or $h > v^2 / g$, the hydraulic jump formula will be applicable or not'). He also showed the existence of critical flow conditions in a rectangular horizontal channel for $V^2 = g\, d$. This was twenty-four and forty-four years respectively before the publications of Ferdinand REECH (1852) and William FROUDE (1872) who were both credited with the introduction of the Reech-Froude number $V / \sqrt{g\, d}$.

Jean-Baptiste BÉLANGER applied successfully the backwater equation upstream and downstream of the hydraulic jump, and pointed out that it cannot be applied across the jump itself (BÉLANGER 1828). He showed also how to estimate the jump location by combining the backwater calculations, upstream and downstream of the jump, with the hydraulic jump equation.

4. Gradually-varied flow calculations: the backwater equation

BÉLANGER (1828) aimed to calculate the free-surface profiles of gradually-varied open channel flows. He developed the backwater equation within a series of basic assumptions. These were: (a) a steady flow, (b) an one-dimensional flow motion, (c) a gradual variation of the wetted surface with distance x along the channel, (d) friction losses that are the same as for an uniform equilibrium flow for the same depth and discharge, and (e) a hydrostatic pressure distribution.

Within the above assumptions, BÉLANGER (1828, pp. 1-11) derived the backwater equation from momentum considerations and he obtained :

$$\sin\theta\,\partial x - \cos\theta\,\partial d - \frac{P_w}{A}(a\,V + b\,V^2) + \frac{Q^2}{g\,A^3}\partial A = 0 \tag{6}$$

where θ is the angle between the bed and the horizontal, x is the longitudinal distance positive downstream, d is the flow depth measured normal to the invert, A is the cross-section area, P_w is the wetted perimeter, Q is the discharge. Equation (6) corresponds to Equation [16] in BÉLANGER (1828, p. 9). It may be rewritten in a more conventional form as a differential equation:

$$\sin\theta - \cos\theta\frac{\partial d}{\partial x} - \frac{P_w}{A}(a\,V + b\,V^2) + \frac{Q^2}{g\,A^3}\frac{\partial A}{\partial x} = 0 \tag{7}$$

In Equations (6) and (7), BÉLANGER (1828) estimated the friction losses using the Prony formula ([6]) :

$$-\frac{\partial H}{\partial x} = \frac{4}{D_H}\left(a\,V + b\,V^2\right) \tag{8}$$

where H is the total head, D_H is the hydraulic diameter: $D_H = 4\,A/P_w$, and a and b are constant. Several values were proposed for the coefficients a and b (App. C). BÉLANGER (1828) used a = 4.44499 10^{-5} and b = 3.093140 10^{-4} (in SI units) that were estimated by Johann EYTELWEIN (1764-1848).

Equation (8) may be compared with modern expressions in terms of the Darcy-Weisbach friction factor:

$$-\frac{\partial H}{\partial x} = \frac{4}{D_H}\left(a\,V + b\,V^2\right) = \frac{f}{D_H}\frac{V^2}{2\,g} \tag{8b}$$

Denoting S_f the friction slope: $S_f = -\partial H/\partial x$, and S_o the bed slope: $S_o = \sin\theta$, BÉLANGER's backwater equation (6) may be combined with the continuity equation to yield:

$$\frac{\partial}{\partial x}\left(d\,\cos\theta + \frac{V^2}{2\,g}\right) = S_o - S_f \tag{9}$$

Equation (9) is essentially identical to modern expressions of the backwater equation (HENDERSON 1966, MONTES 1998, CHANSON 2004). For example, CHANSON (1999) expressed the backwater equation in its most general form as:

$$\cos\theta\frac{\partial d}{\partial x} - d\,\sin\theta\frac{\partial\theta}{\partial x} - \alpha\frac{Q^2}{g\times A^3}\frac{\partial A}{\partial x} = S_o - S_f \tag{10}$$

---

[6] Interestingly BÉLANGER (1849, p. 54) was aware of the work of H.P.G. DARCY (1803-1858) in pipe flows, but he continued to use the Prony formula for its simplicity.

where α is the kinetic energy correction coefficient, or Coriolis coefficient. The main differences between BÉLANGER's equation (9) and Equation (10) are the Coriolis coefficient α and the non-constant bed slope term $d \sin\theta \, \partial\theta/\partial x$. But Jean-Baptiste BÉLANGER made no further assumption and his development (BÉLANGER 1828, p. 9) is basically identical to the modern forms of the backwater equation used by today's hydraulic engineers. BÉLANGER introduced the kinetic energy correction coefficient in a later development of the backwater equation (BÉLANGER 1841, p. 78; 1849, p. 74).

Equation (6) was tested for a non-prismatic smooth drop inlet (Fig. 8). Figure 8A shows the experimental facility and Figure 8B compares the experimental observations with Equation (6) in which the flow resistance was calculated using the Prony formula (Eq. (8)), with Equation (9) in which the friction slope was calculated in terms of the Darcy friction factor, and with Equation (10). All the calculations were performed using the step method, distance calculated from depth. The experimental data (Symbols [*]) are plotted together with the bed elevation z_o and sidewall profiles, and they agree well with the computations (Fig. 8B). The results show basically very little differences between data and calculations, despite the challenging geometry and the crude nature of the Prony formula. BÉLANGER's (1828) calculations give identical results to modern estimates. But Jean-Baptiste BÉLANGER had neither computer nor calculator, nor even slide rule, to integrate the backwater equation. All the calculations were performed manually ([7]), and this explains the common usage of PRONY's simplified formula at the time (BROWN 2002).

Another comparison is presented in Figure 9 for a long prismatic channel. Figure 9A shows some measurements performed by DARCY and BAZIN (1865) in a prismatic, rectangular channel down a steep slope with a downstream control gate. An undular hydraulic jump was observed at x = 125 m. Figure 9B presents the experimental canal along the Canal de Bourgogne and Figure 9C illustrates the measurement technique. In Figure 9A, the free-surface measurements (symbols [*]) are compared with Equation (6) in which the flow resistance was calculated using the Prony formula (Eq. (8)), and with Equation (10) in which the friction slope was calculated in terms of the Darcy-Weisbach friction factor. The location of the hydraulic jump was derived from the application of the momentum principle neglecting the effects of bed slope (Eq. (1)). The results (Fig. 9A) show little differences between Equations (6) and (10). Again, BÉLANGER's (1828) calculations based upon the Prony resistance formula give results close to modern estimates.

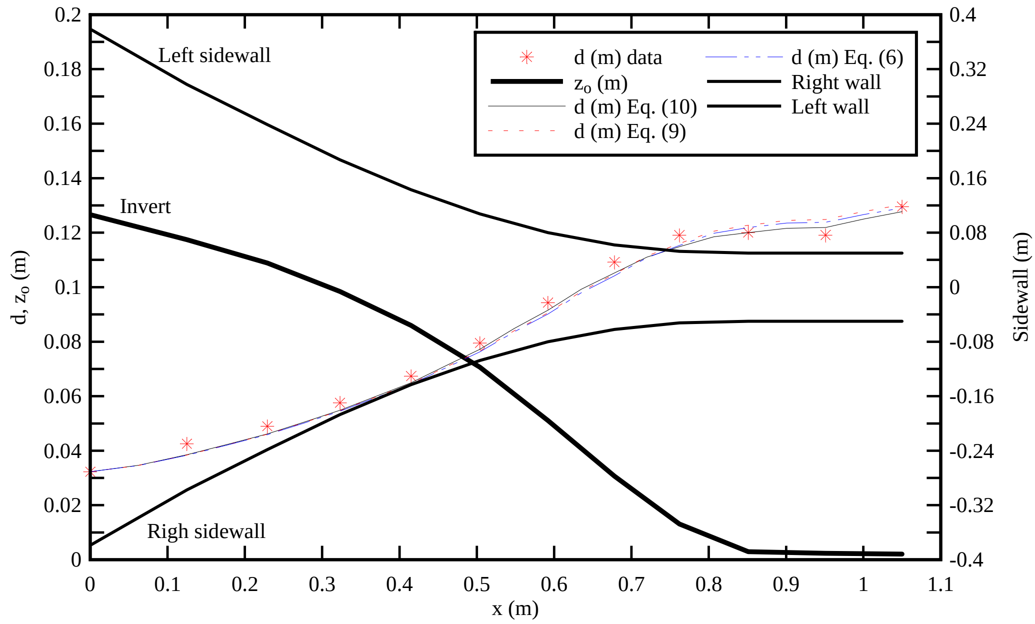
Comments

Jean-Baptiste BÉLANGER integrated the backwater equation by selecting known water depths and calculating manually the distance in between: "*il s'agit d'intégrer entre deux limites h*" ('the integration takes place between two [water depth] limits h') (BÉLANGER 1828, pp. 11-13). Today this technique is called the step method distance calculated from depth (HENDERSON 1966, CHANSON 2004) or the direct step method.

---

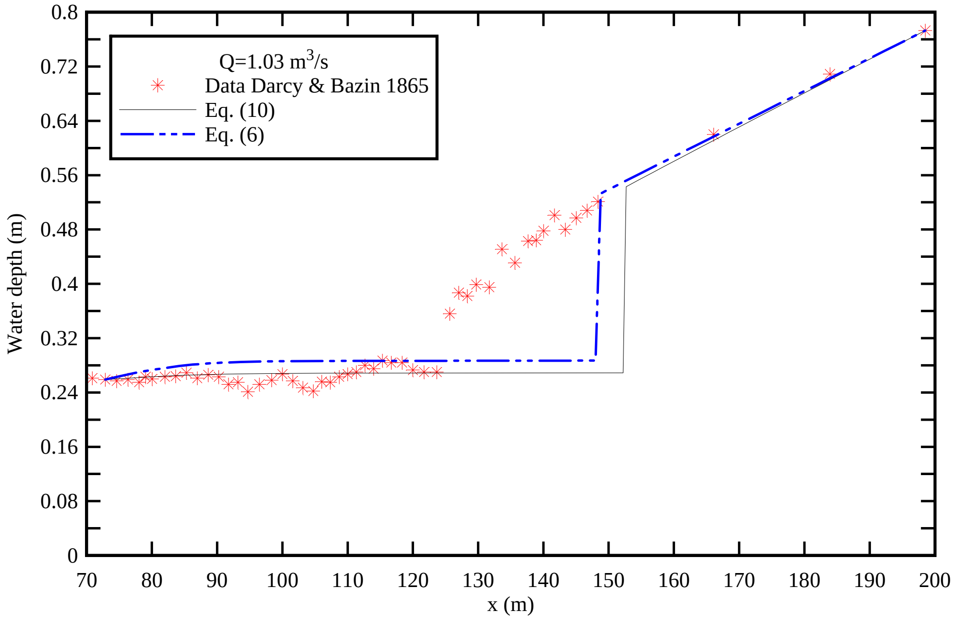
[7] Let us remember that the modern slide rule was introduced in 1859, 31 years later, by the French artillery officer Amédée MANNHEIM (1831-1906).

(A) Photograph of the smooth drop inlet experiment - Flow from bottom right to top left

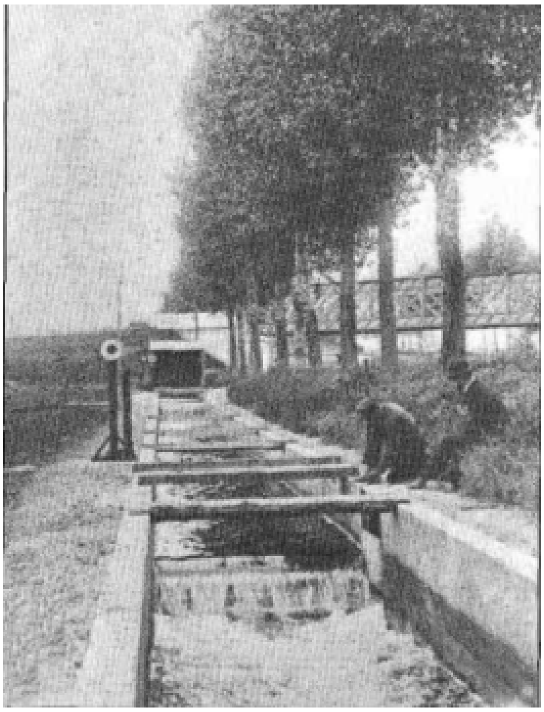


(B) Comparison between experimental data and backwater calculations - Backwater calculations include Equation (6) (BÉLANGER & PRONY), Equation (9) (BÉLANGER & DARCY-WEISBACH) and Equation (10)

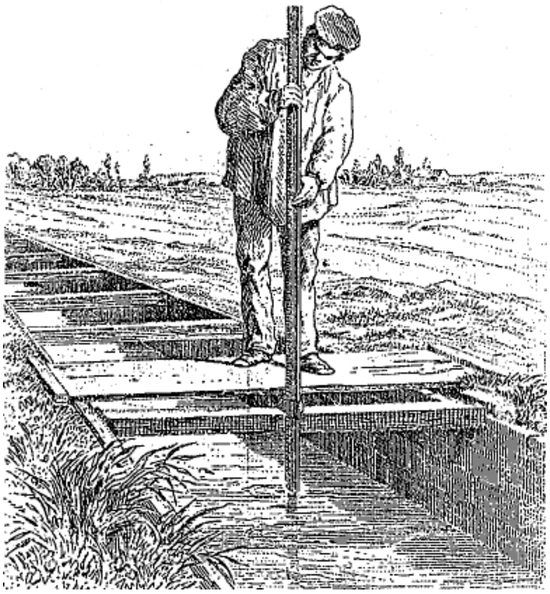
Fig. 8 - Free-surface profile in a smooth drop inlet structure for Q = 0.010 m^3/s



(A) Comparison between backwater calculations and experimental data by DARCY and BAZIN (1865): experimental channel along the Canal de Bourgogne, Q = 1.03 m^3/s, θ = 0.281°, B = 1.99 m, planed boards, series 89 - Backwater calculations include Equation (6) (BÉLANGER & PRONY) and Equation (10)



B) Old photograph of the experimental channel along the Canal de Bourgogne at La Colombière, Dijon (France) (Courtesy of the Centre de Culture Scientifique et Technique de Bourgogne)



(C) Free-surface measurement (DARCY and BAZIN 1865, Plate iv, Fig. 8)

Fig. 9 - Free-surface profile in a long prismatic channel

BÉLANGER further investigated the two singularities of the backwater equation. One corresponded to the uniform equilibrium flow conditions $S_o = S_f$, for which the flow depth equals the normal depth. BÉLANGER (1828, p. 10) obtained the normal depth expression of PRONY (1804):

$$\frac{(a\,V + b\,V^2)}{\frac{D_H}{4}} = \sin\theta \qquad (11)$$

The second singularity of the backwater equation corresponded to $\partial x/\partial d = 0$ and it yielded the condition:

$$\frac{Q^2}{g\,\cos\theta\,A^3}\frac{\partial A}{\partial d} = 1 \qquad (12)$$

that corresponds to the critical flow conditions in a channel of irregular cross-section with hydrostatic pressure distribution. In the particular case of a prismatic rectangular open channel, Equation (12) yields the classical result: $V^2 = g\,d\,\cos\theta$ (LIGGETT 1993, CHANSON 2006). BÉLANGER (1828, p. 29) did not use the term "critical flow" but he highlighted explicitly the flow singularity: "*un cas peu ordinaire*" ('a special case'). He stressed further the physical impossibility to observe $\partial d/\partial x = +\infty$ for this 'special case'.

5. Discussion

Twenty one years after this original essay, BÉLANGER (1841) expanded his treatment in the form of a series of lecture notes for the Ecole des Ponts et Chaussées ([8]) for the session 1841-1842. His notes formed a

[8] The lecture notes were used at the Ecole des Ponts et Chaussées and Ecole Centrale des Arts et Manufactures, and available at the Ecole Polytechnique et Ecole des Mines de Paris.

comprehensive treatise in hydraulic engineering, and they were re-edited several times, while Jean-Baptiste BÉLANGER was lecturing, with relatively small to moderate differences between the various editions: e.g., BÉLANGER (1841,1849) for the university sessions 1841-1842 and 1849-1850 at the Ecole des Ponts et Chaussées respectively.

On the hydraulic jump, Jean-Baptiste BÉLANGER (1849) indicated that he found his error in 1838. He corrected his treatment of the hydraulic jump and applied correctly the momentum principle, "*le théorème relatif à l'accroissement de la quantité de mouvement*" ('the theorem related to the rate of increase in momentum') (BÉLANGER 1841, p. 87). In his derivation, he stated : "*l'accroissement algébrique de la quantité de mouvement [...] est égale à la somme des impulsions des forces extérieures, projetées parallèlement au mouvement*" ('the increase in momentum [...] is equal to the sum of external forces projected in the flow direction') (BÉLANGER 1849, p. 85). His newer result yielded the "modern" form of the Bélanger equation :

$$\frac{d_2}{d_1} = -\frac{1}{2} + \sqrt{\frac{1}{4} + 2\,\alpha'\,Fr_1^2} \tag{5a}$$

that is basically a rewriting of Equation (1). For a hydraulic jump in a horizontal, rectangular and prismatic channel, BÉLANGER (1849, p. 88) calculated the loss in kinetic energy head :

$$\frac{V_1^2}{2\,g} - \frac{V_2^2}{2\,g} = \frac{(d_1 + d_2)^2}{4\,d_1 d_2}\,(d_2 - d_1) \tag{13}$$

that may be rewritten in terms of the head loss in the hydraulic jump:

$$\Delta H = \frac{(d_2 - d_1)^3}{4\,d_1 d_2} \tag{14}$$

Equation (14) is a well-known result for a hydraulic jump in a horizontal rectangular channel (HENDERSON 1966, MONTES 1998, CHANSON 2004).

In the same treatise, BELANGER (1841,1849) presented explicitly a number of basic features of open channel flows. He developed an expression of the uniform equilibrium flow depth (normal depth) that was derived from energy considerations. He further developed the calculations of the normal depth for a composite channel (Fig. 10), showing accurately that the total discharge is the sum of the flow rates in the main channel and in the flood plain, and that the friction slope is identical for both channel sections, but with different friction coefficients.

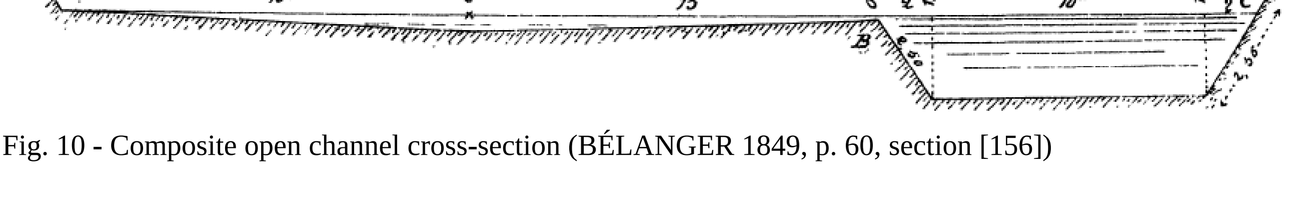


Fig. 10 - Composite open channel cross-section (BÉLANGER 1849, p. 60, section [156])

BELANGER (1841,1849) showed that, in a rectangular channel, the discharge per unit width is maximum at critical flow conditions for a given specific energy E: "*entre ces deux valeurs de ξ, il y en une pour laquelle Q devient maximum [...] quand on a ζ = 1/3 Z*" ('between these two values if ζ, there is one for which Q is maximum [...] when ξ = 1/3 Z'). He derived the expression of the critical depth d_c :

$$d_c = \frac{2}{3} E \qquad (15)$$

Equation (15) was obtained in section [85] (BELANGER 1849, p. 33) as part of a discussion of the overflow on a broad-crested weir (Fig. 11). His treatment of the broad crested weir yielded further the classical expression of the flow rate Q:

$$\frac{Q}{B} = \sqrt{g}\left(\frac{2}{3} H\right)^{3/2} \qquad (16)$$

for a rectangular channel of width B, where H is the total head above the crest invert.
All these results are common knowledge today (HENDERSON 1966, CHANSON 2004), but were new and important developments in the 1840s.
It is worth noting that BÉLANGER (1841,1849) used both momentum and energy considerations in a somewhat inconsistent manner. Such inconsistencies were discussed by YEN (2002) and CHANSON (1999,2004) in a broader context. In his treatment of the hydraulic jump, BÉLANGER (1841,1849) used correctly the momentum principle, but introduced a kinetic energy correction coefficient. Similarly he solved the uniform equilibrium flow based upon energy considerations, although modern treatments are derived from the momentum equation. Late Professor Ben YEN pointed accurately a number of similar discrepancies in recent studies by some hydraulic engineers (YEN 2002).

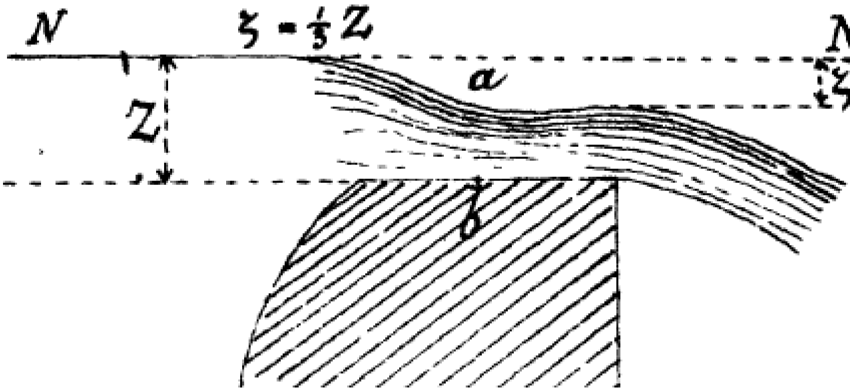


Fig. 11 - Flow over a broad-crested weir (BÉLANGER 1849, p. 33, section [85])

6. Conclusion

In the 1820s, Jean-Baptiste BÉLANGER (1790-1874) worked on a method to calculate gradually-varied open channel flow properties for steady flow conditions. Although he succeeded, his treatise (BÉLANGER 1828) is sometimes known for the treatment of the stationary hydraulic jump, nowadays called the Bélanger equation. It is shown herein that BÉLANGER (1828) correctly considered a hydraulic jump as a rapidly-varied flow, but he applied the wrong basic principle at the time. The error was corrected ten years later and the correct solution was first published by BÉLANGER (1841).

The originality of BÉLANGER's (1828) work was the successful development of the backwater equation for steady one-dimensional gradually-varied flows in an open channel. His work outlined the fundamental assumptions and he derived from momentum considerations an equation for gradually-varied open channel flows that is still in use today, but for the flow resistance model. In the same study, Jean-Baptiste BÉLANGER introduced two further modern concepts: the step method, distance calculated from depth, and the critical flow conditions. He associated the notion of critical flow with one of the two singularities of the backwater equation. His technique of numerical integration was ahead of his time, when there was no computer nor electronic calculator.

In 1828, Jean-Baptiste BÉLANGER was a young hydraulic engineer and his contribution (BÉLANGER 1828) demonstrates the dynamism of practicing engineers at the time. In the second part of his career, Jean-Baptiste BÉLANGER became a renowned academic at the leading French engineering schools. He continued to work in open channel hydraulics including on the hydraulic jump (BÉLANGER 1841). His contributions were remarkable and influenced other leading hydraulic engineers including BRESSE (1860), DARCY and BAZIN (1865), BARRÉ de SAINT VENANT (1871), and BOUSSINESQ (1877).

7. Acknowledgements

The writer thanks Dr Jerry R. ROGERS, University of Houston for his detailed review of the report, valuable comments and encouragements. He acknowledges the helpful comments of many people, including Dr Jerry L. ANDERSON, University of Memphis, Dr Glenn O. BROWN, Oklahoma State University, Professor Colin J. APELT, University of Queensland, and Professor John D. FENTON, University of Karlsruhe.

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Appendix A - Birth certificate of Jean-Baptiste BÉLANGER

The Archives Municipales de Valenciennes provided the author with a copy of the birth certificate ([1]) of Jean-Baptiste BÉLANGER dated 4 April 1790 (Fig. C-1), together with a facsimile of the marriage certificate of his parents dated 21 April 1789. The birth certificate was handwritten and the French text is reproduced after Figure C-1, with the same spelling, punctuation and style as the original document. Jean-Baptiste BÉLANGER was baptised on the day of his birth (4 April 1790). His godfather and godmother were respectively his uncle and his aunt.

The facsimile of his parents' marriage certificate is also reproduced, with the same spelling, punctuation and style as the original facsimile (Dated 26 June 2008).

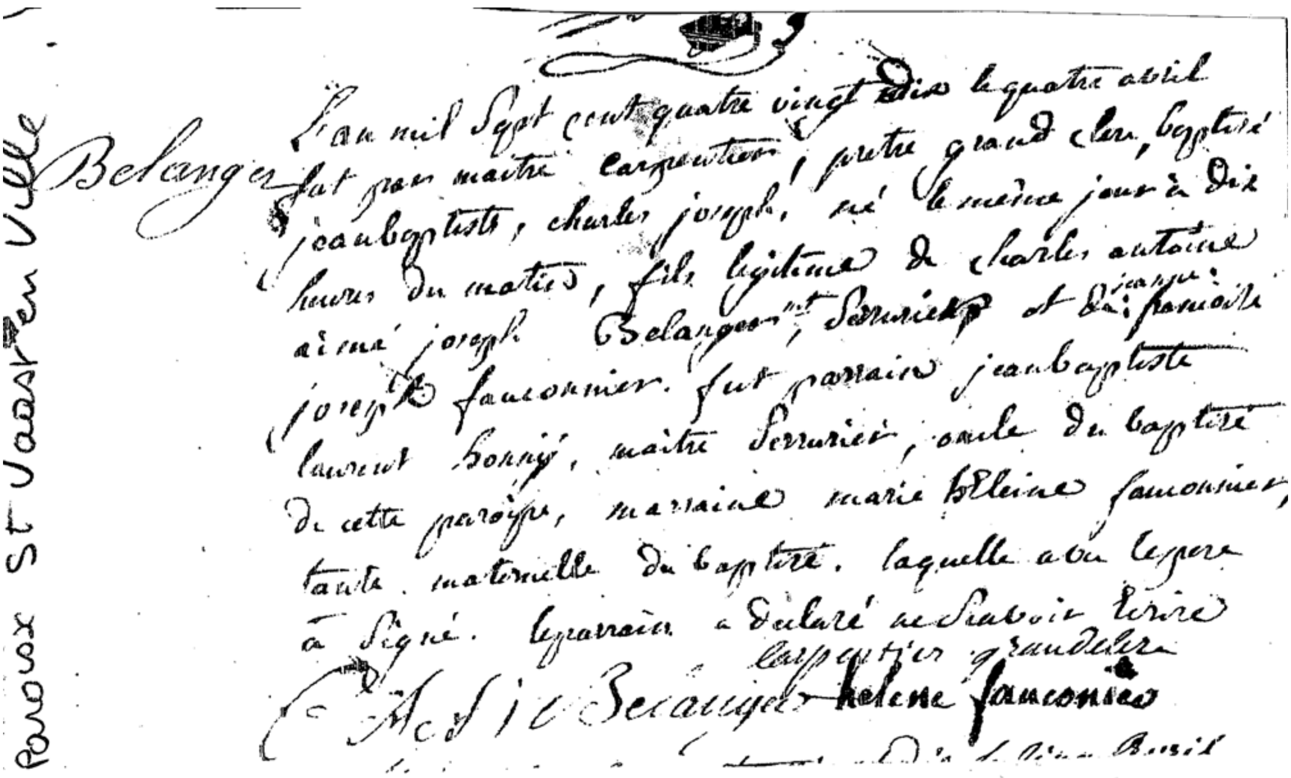

Fig. C-1 - Birth certificate of Jean-Baptiste BÉLANGER, paroisse de St Vaast en Ville (Courtesy of Archives Municipales de Valenciennes)

Text

Belanger	L'an mil Sept cent quatre vingt dix le quatre avril fur par maitre Carpentier, pretre grandeler, baptisé jeanbaptiste, charles joseph, né le même jour à Dix heures du matin, fils légitime de Charles antoine

[1] The birth certificate acted as both birth and baptism certificates since it was common that the newborns were baptised on the day of their birth by the local catholic priest.

	aimé joseph Belanger, Serurrier et de jeanne françoise joseph fauconnier. fut parrain jean baptiste laurent honnis, maitre Serurrier, oncle du batisé de cette paroisse, marraine marie heleine fauconnier, tante maternelle du baptisé. laquelle avec le pere a Signé. le parrain a déclaré ne Savoir lire Carpentier grandeler Charles Belanger Helene fauconnier

Note : blue text = uncertain letters/words.

Facsimile of marriage certificate of Charles Antoine Aimé Joseph BÉLANGER and Jeanne Françoise Joseph FAUCONNIER on 21 April 1789

Extrait d'acte de mariage

Acte rédigé le : 21/04/1789 par: Maitre GOPEAU (Curé) Célébrant / Officier d'état civil

à VALENCIENNES, Paroisse/Canton: St Géry

Concernant le mariage

de:	Charles Antoine Aimé Joseph BELLANGER
Agé(e) de:	26 ans et 3 mois (natif de la paroisse de St Etienne à Lille
Exerçant la profession de :	Serurrier
Fils / Fille de :	feu Antoine Joseph
et de :	Angélique GODEFROY
et de :	Jeanne Françoise Joseph FAUCONNIER
Agé(e) de :	27 ans (native de cette paroisse)
Exerçant la profession de :	
Fils / Fille de :	Pierre Joseph
et de :	Marie Anne BERTON
1er témoin:	François FAUCONNIER
Parenté:	Frère de l'épouse
2ème témoin:	Jean-Baptiste HONNIS
Exerçant la profession de :	Serurrier

Fait à Valenciennes, le 26/06/08

Appendix B - The names of the 72 scientists written around the Eiffel Tower, Paris

The names of 72 scientists were engraved around the first floor of the Eiffel Tower (Tour Eiffel, Paris) as a tribute gesture from Gustave EIFFEL (Fig. A-1 and A-2). The names disappeared during one of the Eiffel Tower's repaintings at the turn of the 20th century and they were re-established in 1986-87.

No	Name	No	Name	No	Name	No	Name
1	Marc SÉGUIN (1786-1875)	19	Jules Célestin JAMIN (1818-1889)	37	Augustin Louis CAUCHY (1789-1857)	55	Jules Alexandre PETIET (1813-1871)
2	Joseph Jérôme LEFRANCOIS de LALANDE (1732-1807)	20	Louis Joseph GAY-LUSSAC (1778-1850)	38	Eugène BELGRAND (1810-1878)	56	Louis Jacques MANDÉ DAGUERRE (1787-1851)
3	Henri TRESCA (1814-1885)	21	Hippolyte FIZEAU (1819-1896)	39	Henri Victor REGNAULT (1810-1878)	57	Charles Adolphe WÜRTZ (1817-1884)
4	Jean-Victor PONCELET (1788-1867)	22	Eugène SCHNEIDER (1805-1875)	40	Augustin Jean FRESNEL (1788-1827)	58	Urbain Jean Joseph LE VERRIER (1811-1877)
5	Jacques Antoine Charles BRESSE (1822-1883)	23	Louis LE CHATELIER (1815-1873)	41	Gaspard de PRONY (1755-1839)	59	Jean Albert Vincent Auguste PERDONNET (1808-1867)
6	Joseph-Louis LAGRANGE (1736-1813)	24	Pierre BERTHIER (1782-1861)	42	Louis VICAT (1786-1861)	60	Jean-Baptiste Joseph DELAMBRE (1749-1822)
7	Jean-Baptiste-Charles-Joseph BÉLANGER (1790-1874)	25	Jean-Augustin BARRAL (1819-1884)	43	Jacques-Joseph EBELMEN (1814-1852)	61	Étienne Louis MALUS (1775-1812)
8	Baron Georges Leopold Chretien Frédéric Dagobert CUVIER (1769-1832)	26	Henri de DION (1828-1878)	44	Charles-Augustin de COULOMB (1736-1806)	62	Louis BREGUET (1804-1883)
9	Pierre-Simon LAPLACE (1749-1827)	27	Ernest GOUIN (1815-1885)	45	Louis POINSOT (1777-1859)	63	Camille POLONCEAU (1778-1847)
10	Pierre Louis DULONG (1785-1838)	28	Louis Didier JOUSSELIN (1776-1838)	46	Jean Bernard Léon FOUCAULT (1819-1868)	64	Jean Baptiste André DUMAS (1800-1884)
11	Michel CHASLES (1793-1880)	29	Paul Pierre BROCA (1824-1880)	47	Charles-Eugène DELAUNAY (1816-1872)	65	Benoît Paul Émile CLAPEYRON (1799-1864)
12	Antoine Laurent de LAVOISIER (1743-1794)	30	Antoine BECQUEREL (1788-1878)	48	Arthur MORIN (1795-1880)	66	Jean-Charles de BORDA (1733-1799)
13	André-Marie AMPÈRE (1775-1836)	31	Gaspard-Gustave CORIOLIS (1792-1843)	49	René Just HAÜY (1743-1822)	67	Jean Baptiste Joseph FOURIER (1768-1830)
14	Michel Eugène CHEVREUL (1786-1889)	32	Jean-François CAIL (1804-1871)	50	Charles COMBES (1801-1872)	68	Marie François Xavier BICHAT (1771-1802)
15	Eugène FLACHAT (1802-1873)	33	Jacques TRIGER (1801-1867)	51	Louis Jacques THÉNARD (1777-1857)	69	François Clément SAUVAGE (1814-1872)
16	Claude Louis Marie Henri NAVIER (1785-1835)	34	Henri GIFFARD (1825-1882)	52	Dominique François Jean ARAGO (1786-1853)	70	Théophile-Jules PELOUZE (1807-1867)
17	Adrien-Marie LEGENDRE (1752-1833)	35	François PERRIER (1833-1888)	53	Siméon Denis POISSON (1781-1840)	71	Lazare Nicolas Marguerite CARNOT (1753-1823)
18	Jean-Antoine CHAPTAL (1756-1832)	36	Jacques Charles François STURM (1803-1855)	54	Gaspard MONGE (1746-1818)	72	Gabriel LAMÉ (1795-1870)

The names numbered between 1 and 18 are on the Facade Trocadéro. The names numbered from 19 to 36 are engraved on the Facade Grenelle. The name numbers 37 to 54 are written on the Facade Ecole Militaire, while names numbered between 55 and 72 are listed on the Facade Paris.

Fig. A-1 - Eiffel Tower (Tour Eiffel), Paris, France on 8 July 2008 (Courtesy of Bernard CHANSON) - View from Champ de Mars (Facade Ecole Militaire) with the Trocadéro in the background

(A) Facade Ecole Militaire with the names of CAUCHY, BELGRAND, REGNAULT, FRESNEL, de PRONY, and VIGAT

(B) Facade Trocadéro with the names of PONCELET (incomplete), BRESSE, LAGRANGE, BÉLANGER, CUVIER, and LAPLACE

(C) Facade Ecole Militaire with the names of COMBES (incomplete), THENARD, ARAGO, POISSON and MONGE

Fig. A-2 - Details of the engraved names of scientists on the Eiffel Tower on 25 July 2008

Appendix C - The Prony flow resistance formula

Gaspard Clair François Marie Riche de PRONY (1755-1839) introduced a flow resistance formula for pipes and rivers composed of a linear and a quadratic velocity term (PRONY 1804). The Prony formula was originally presented as :

$$\frac{1}{4} g \, S_f \, D_H = a' \, V + b' \, V^2 \qquad \text{(B-1)}$$

where g is the gravity acceleration, D_H is the hydraulic diameter: $D_H = 4 \, A/P_w$, A is the cross-section area, P_w is the wetted perimeter, S_f is the friction slope: $S_f = -\partial H/\partial x$, H is the total head, x is the longitudinal direction positive downstream, V is the cross-sectional averaged velocity, and a' and b' are constant. The Prony formula was typically used in the form:

$$S_f = \frac{4}{D_H} \left(a \, V + b \, V^2 \right) \qquad \text{(B-2)}$$

Several values were proposed for the coefficients a and b (Table B-1). However the differences were relatively small as shown in Figure B-1, with discrepancies between the formulae of less than 3% for V = 1 m/s.

Despite its crude nature, the Prony formula gave results close to modern estimates. Its success may be explained by its simple form at a time when all the calculations were performed manually (BROWN 2002).

Table B-1 - Coefficients a and b of the Prony formula (Eq. (B-2))

Reference / Researcher	a	b
Open channel flows		
PRONY (1804, p. xxix)		
31 experiments (0.03 < V < 2.3 m/s)	$4.4444 \ 10^{-5}$	$3.0928 \ 10^{-4}$
BÉLANGER (1828, p. 6)		
Gaspard Clair François Marie Riche de PRONY (1755-1839)	$4.445 \ 10^{-5}$	$3.09314 \ 10^{-4}$
Johann EYTELWEIN (1764-1848)	$2.4265 \ 10^{-5}$	3.655410^{-4}
BÉLANGER (1841, pp. 66-67; 1849, p. 67)		
Gaspard Clair François Marie Riche de PRONY (1755-1839)	$4.44 \ 10^{-5}$	$3.09 \ 10^{-4}$
Johann EYTELWEIN (1764-1848)	$2.4 \ 10^{-5}$	$3.66 \ 10^{-4}$
Circular pipe flows		
PRONY (1804, p. xxix)		
51 experiments (0.03 < ∅ < 0.5 m)	$1.7329 \ 10^{-5}$	$3.4822 \ 10^{-4}$
BÉLANGER (1849, p. 49)		
Gaspard Clair François Marie Riche de PRONY (1755-1839)	$1.73 \ 10^{-5}$	$3.48 \ 10^{-4}$
Johann EYTELWEIN (1764-1848)	$2.22 \ 10^{-5}$	$2.80 \ 10^{-4}$
Jean-François D'AUBUISSON (1769-1841),	$1.884 \ 10^{-5}$	$3.425 \ 10^{-4}$

Note: SI units.

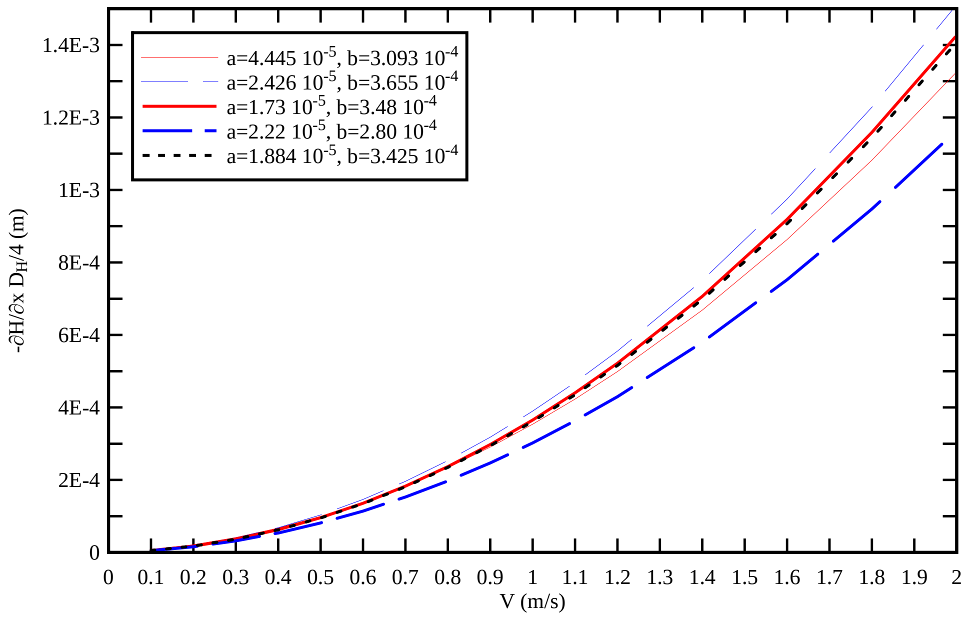

Fig. B-1 - Prony flow resistance formula: comparative results between various values of the coefficients a and b

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