CHAPTER I

INTRODUCTION TO THE STUDY OF FLUID MOTION

1. MECHANICS OF FLUIDS AS AN ENGINEERING SCIENCE

Historical development. Some two hundred years ago, mankind's centuries of experience with the flow of water began to crystallize in scientific form. Despite this common origin, however, two distinct schools of thought gradually evolved. On the one hand, through the convenient creation of an "ideal fluid," mathematical physicists developed the theoretical science known as *classical hydrodynamics.* On the other hand, claiming that idealized theories were of no practical use without empirical correction factors, engineers developed from experimental findings the applied science known as *hydraulics*, for the specific fields of water supply, irrigation, river control, and water power.

As other engineers became confronted with problems involving the flow of oil and gas in pipes or of air in ventilating systems, hydraulics formulas were gradually extended to fill these needs. The design of modern aircraft, on the contrary, permitted from the outset much smaller "factors of ignorance" than most engineering fields. Not only were existing principles of hydraulics often too inexact for use in aeronautics, but these principles were not sufficiently descriptive of fluid motion in general to be adaptable to the motion of bodies through air. Fortunately, rather than develop purely empirical methods to supply this need, physicists and engineers began instead to expand the basic concepts of fluid motion into a science which now includes as specialized phases both aerodynamics and hydraulics, as well as a host of other applied fields.

This science has come to be known as the *mechanics of fluids*, a subject parallel to the mechanics of solids and engineering materials and built upon the same fundamental laws of motion. Unlike empirical hydraulics, therefore, it stems primarily from basic physical principles; unlike the purely mathematical treatment of classical hydrodynamics, on the other hand, the science is closely correlated with experimental studies which both complement and substantiate the fundamental analysis. In a word, the mechanics of fluids avoids the weaknesses of its forebears, but draws heavily upon the strength of each for much of its material.

Present-day scope. So broad in scope is this relatively new science that it forms a basis for such varied fields as meteorology, oceanography, ballistics, lubrication, marine engineering, and even certain phases of geology—not to mention hydraulic engineering and aeronautics. In fact, a single example, the phenomenon known as turbulence, will suffice to show the almost universal importance of fluid motion to present-day civilization. Turbulence, or the presence of eddies in a moving fluid, gives rise to the twofold effect of a pronounced mixing of the fluid and a subsequent dissipation of energy. Like solid friction, fluid turbulence is sometimes a blessing to mankind, and sometimes a curse. Without the mixing which eddies produce, both the water in boiler tubes and the air surrounding the earth would be very poor distributors of heat; steam engines would prove too costly to run, and the atmosphere would be incapable of supporting life. On the other hand, dust storms would not then occur, nor would rivers transport their tremendous loads of silt from the foothills to the sea. Without means of producing eddies in the process of propulsion, moreover, a swimmer—or an ocean liner—could make little headway, just as a car would remain at rest if friction provided no tractive force. Yet, paradoxically, were turbulence not produced by the motion of a body through a fluid, the process of streamlining would be quite unnecessary. Even the act of breathing depends upon turbulence, for without the violent mixing which accompanies exhalation no fresh air could be inhaled thereafter unless one moved to a new location. If but one aspect of fluid motion can be of such general importance, it should be evident that the principles of fluid motion as a whole must govern a vast realm of human endeavor. A clear understanding of these principles is therefore essential to the modern scientist and engineer.

2. FUNDAMENTAL CHARACTERISTICS OF FLOW

Units of measurement. Fluid motion, like other phases of mechanics, may fully be described in units of *length, time,* and *force.* For instance, the form of an observation or barrage balloon and its elevation above the earth can be specified in detail solely in terms of the length dimension; the length and time dimensions together provide a basis for expressing its rate of ascent and the wind velocity encountered at any elevation; and the lift of the balloon and the drag of balloon and cable, in units of force, complete the description of the practical features of the motion. Likewise, the shape of a lubricated shaft and bearing can be stated in terms of geometrical measurements, the rotational speed in terms of time, and the shear and pressure of the fluid lubricant in terms of length and force. Once such flow characteristics are known, any problem of fluid motion is, for all practical purposes, completely solved.

There are fairly simple means of measuring length, time, and force—whether individually or in combination—in laboratory, shop, and field, so that the characteristics of practically any kind of fluid motion may be determined either directly or indirectly. The measurement of boundary form, as the reader is well aware, requires in effect only a protractor and a linear scale, whether one deals with the minute fractions of an inch involved in bearing clearances or with the thousands of feet over which atmospheric disturbances may extend. The basic unit of time is generally considered to be the second, and such short-period phenomena as the induced vibration of a submarine periscope may be of this order of magnitude; on the other hand, flood movements in rivers may be followed more conveniently with a calendar than with a stop watch, while tides have daily, monthly, and even seasonal cycles. Force, normally measured with a calibrated spring, likewise varies greatly in magnitude from one flow phenomenon to the next, the resistance to the settling of dust particles in air being a minute fraction of an ounce as compared to the thrust of many tons exerted by the screws of an ocean liner.

The measurement of fluid *velocity*, a combination of length and time, is somewhat more involved. For example, one may time the movement of a float over a known distance, or count the revolutions per minute of a sensitive propeller which has been calibrated in a flow of known speed. Through simultaneous use of many such floats or current meters, moreover, one can obtain a picture of the rate and direction of movement over a large region, as in maps of prevailing winds or ocean currents. Another combination of length and time units is illustrated by *volume rate of flow:* the number of drops of liquid emerging per second from a pipette, or the number of million gallons of water passing per day through the supply mains of a large city.

Quite as often as one measures the force exerted upon a body by a fluid, one is interested in determining the *intensity* of such force at a given locality. The local intensity of the *pressure* exerted upon a body by the atmosphere, for instance, is usually measured in pounds per square inch, although far below the ocean surface, or in high-pressure equipment, the intensity of pressure is so great that it is often evaluated in "atmospheres," or multiples of the normal atmospheric intensity. Dimensionally similar to pressure intensity is the intensity of *shear*, such as the unit tangential stress exerted by the moving fluid on a lubricated bearing or on the wall of a pipe.

Problems of flow prediction. It should be evident to the reader that the correct measurement of such flow characteristics is very essential in the study of existing states of fluid motion—for instance, in checking the efficiency of a hydraulic turbine or testing the performance of a new method of fluid transmission. Nevertheless, flow measurement is not the ultimate goal of the modern mechanics of fluids, but merely a necessary tool, for science and engineering today require not so much the measurement as the accurate prediction of one or another characteristic of motion from known or assumed conditions. For example, in designing a plumbing system for a large building it is not sufficient to note after installation if, and at what rate, water is available at the top floor; the system must be so designed prior to construction that the required flow can be guaranteed, yet without waste of material or power. Similarly, the drag and lift of a model airplane may easily be measured in a wind tunnel, but only if the results are properly converted to the actual scale of the prototype will the tests have any practical significance. Atmospheric pressures and wind velocities, likewise, can readily be recorded during a storm, but a meteorologist is not worth his salt if he cannot foretell such conditions of motion days in advance. In the large-scale control of rivers, in the streamlining of high-velocity craft, in submarine signaling and detection—in fact, in any problem which involves the motion of a fluid medium—it is obviously necessary to understand the essential principles of such motion in order to predict with fair certainty what the characteristics of flow will be under any given conditions.

3. FLUID PROPERTIES AS A GUIDE TO STUDY

Mechanical properties of fluids. Were the conditions of motion of different fluids completely unrelated, the task of flow analysis would indeed be hopeless. All fluids, however, possess essentially the same mechanical properties, differing from one another only in degree. Water, for example, is well known to be less dense than mercury but considerably more dense than air; it follows that the inertial characteristics of water will be less pronounced than those of mercury and more pronounced than those of air—in other words, in comparison with water it should be relatively simple to set air in motion but relatively difficult to produce the same rate of motion in mercury. The basic role of *mass* in resisting such acceleration is nevertheless identical in all three instances, and is embodied in a single principle of fluid mechanics.

Water, furthermore, is known to flow down a slope as a result of *weight*, or gravitational attraction. Few realize, however, that a relatively slight difference in fluid weight due to local temperature changes in the atmosphere or in the ocean will likewise result in the "downhill" flow of the colder within the warmer medium. The principle of gravitational action in river hydraulics thus has its counterpart in meteorology and oceanography, to mention only two of many fields.

Likewise, although the *viscosity* of lubricating oil is obviously far higher than that of alcohol, tests on the flow of oil through a pipe may, through present-day knowledge, be used to predict with great accuracy the corresponding flow characteristics of alcohol—or of steam, or natural gas—because the principle of viscous resistance is the same for any true fluid. Indeed, the wind resistance of skyscrapers or suspension bridges could just as well be investigated on models towed through water as on models held in a stream of air.

Gases, finally, are well known to be readily compressible, but liquids are normally considered of fairly constant density even under considerable pressure. Yet the fact that the *elastic modulus* of water, for instance, is well below that of steel indicates that it is also compressible, even though to a much lower degree than air. As a result, the propagation of elastic waves in the sea follows quite the same basic laws as the propagation of sound in the atmosphere, however much the actual velocity of propagation may differ. Indeed, the analogy may even be extended to include other fluid properties, for the *capillary* waves in front of a model bridge pier, the *gravity* waves at the bow of a ship, and the *sound* waves at the nose of a projectile may be analyzed through essentially the same principles of fluid mechanics.

Influence of fluid properties upon flow characteristics. Before proceeding to the study of each fluid property, it is necessary for the reader to become familiar with methods of visualizing a *flow pattern*, such as that produced, let us say, by the speeding of an automobile along a highway or by the flow of water over a spillway. As a first approximation, such patterns of fluid motion may be obtained directly from the geometrical form of the flow boundaries, and in the following chapter will be found a detailed treatment of the corresponding velocities and accelerations without consideration of the fluid forces which are involved. Fluid density is introduced in the chapter thereafter, but the sole force discussed is that due to the pressure variation which accompanies the acceleration of a fluid according to the various patterns of motion already studied.

Were fluid properties other than density non-existent, as is conveniently assumed in classical hydrodynamics, further study would be unnecessary. As a matter of fact, the principles developed in these earlier chapters will be found sufficient for the solution of a number of practical flow problems. But in order to recognize these problems, and as well to deal effectively with the many which do not fall in this category, one must study in detail the effect of each additional fluid property upon the basic patterns of motion already discussed.

Therefore, the influence of fluid weight, or gravitational attraction, upon the distribution of pressure and velocity is then described for the same elementary boundary forms, whereafter use is made of a simplified method of analyzing flow in conduits and open channels. Fluid viscosity is next introduced, leading directly to the principles of fluid turbulence, boundary resistance, lift, and propulsion, which play extremely important roles in present-day analysis. The influence of surface tension, perhaps the least important of the fluid properties, is then briefly discussed. Finally, the effect of compressibility upon the flow pattern is described, with particular attention to motion at velocities greater than that of sound; since this phase of fluid mechanics verges upon the realm of *thermodynamics*, only those phenomena are treated which bear a close relationship to the earlier portions of the text.

As is true in all phases of mechanics, the basic principles of fluid motion are subject to mathematical formulation as well as physical description. The principles considered essential in the present book are therefore derived through use of the elementary calculus required of all undergraduate engineers, and every derivation is accompanied by a detailed physical interpretation. Despite the resulting precision of each individual principle, however, it is generally difficult and often impossible to combine these principles in the rigorous solution of flow problems involving the influence of more than one fluid property. As the reader advances, he will accordingly become aware of the extent to which engineers and scientists must still use simplifying approximations in the final analysis of fluid motion. For this reason, if for no other, satisfactory results must invariably depend upon a sound grasp of the fundamental principles herein derived.

QUESTIONS FOR CLASS DISCUSSION

1. Under what circumstances did man probably first realize a need for knowledge of the basic principles of fluid motion?

2. Cite examples of flow phenomena encountered in (*a*) everyday life and (*b*) engineering practice.

3. Suggest how an understanding of fluid behavior might advantageously be applied in the following fields: bridge design; oil-well drilling; sanitary engineering; mining and metallurgy; soil conservation; grading of abrasives; chemical engineering; geology.

4. Is the motion of the atmosphere always turbulent? By what visual means readily at hand may such motion be observed?

5. Why are modern vehicles streamlined? Cite cases in which streamlining is (*a*) scientifically sound and (*b*) merely aesthetic.

6. Is man's blood stream probably turbulent like the flow through a moderately large boiler tube, or is it more reasonable to conclude that it exemplifies another means of transferring heat?

7. High winds tend to lift roofs from buildings rather than to force them inward. Explain this effect in terms of pressure distribution.

8. Suggest a likely cause of the "singing" of telephone wires in the wind.

9. Enumerate the fluid properties which can influence the behavior of a fluid in motion. Suggest flow phenomena illustrating the influence of each property.

10. It is a well-known fact that one can float more easily in salt water than in fresh. Should one also be able to swim faster in salt water?

11. Should mercury or water be expected to flow more rapidly down a channel of given slope?

12. Why is it necessary in winter to use a "lighter" oil for automobiles than in summer? To what property does the term "light" refer?

13. Would you imagine the viscous resistance to the flow of air to be greater or less than the viscous resistance to the flow of water?

14. Is the pressure intensity (*a*) within a bubble of gas, and (*b*) within a drop of liquid, probably greater than, equal to, or less than that of the surrounding medium?

15. Projectiles are known to whistle or scream during flight. Should the sound wave follow a projectile or precede it?

16. *Gulliver's Travels* is sometimes criticized on the basis of conversion principles now used in model experiments. Should gravitational attraction seem relatively greater to a Lilliputian or to a Brobdingnagian? To which would the day seem shorter? To which would water appear more viscous? Which would have less difficulty with surface tension? Assuming a height ratio of 1 : 100, estimate their relative weights, velocities of walking, and frequencies of breathing.

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CHAPTER II

FLUID VELOCITY AND ACCELERATION

4. VELOCITY AND THE STREAM LINE

Visualization of the flow pattern. Smoke emerging from a chimney on a windy day permits a fascinating visual study of the motion of the surrounding air. The movement of the atmosphere on a larger scale is likewise made visible by the behavior of clouds, and currents in a

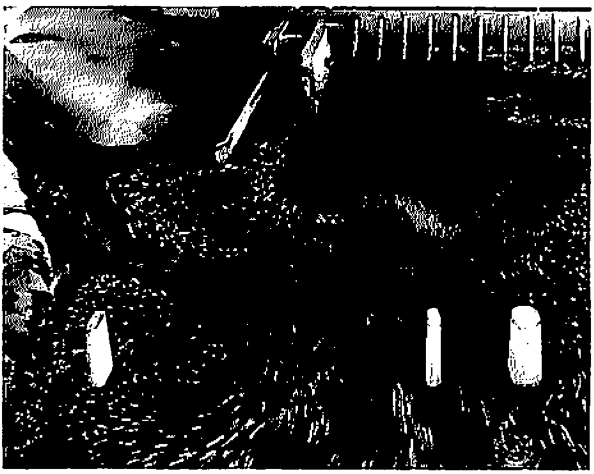


PLATE I. Currents in a river model shown by the motion of confetti scattered on the water surface; from a navigation study by the U. S. Engineer Sub-Office in the Hydraulics Laboratory at Iowa City.

river or canal by the silt and detritus carried in suspension. In each of these examples, one can either watch the trend of the movement in a given zone, or else choose at random a small portion of the suspended matter and follow it along its path, thereby obtaining a mental record of particular phases of the motion.

In the majority of flow phenomena, however, such visible agents as soot and silt are not so conveniently suspended in the moving fluid.

In the laboratory, to be sure, it is common practice to introduce droplets of oil or shiny particles of aluminum for visual or photographic observation, but for general purposes one's only recourse is to construct on paper or in the imagination a system of flow lines showing the nature of the motion in any desired region. Such a system of lines may seem at first quite a haphazard affair; the motion which it indicates, nevertheless, must be in complete accord with principles of mechanics, the investigation of which is the purpose of this book. While the ultimate goal of the present chapter is the determination of the flow pattern around or between boundaries of any given form, it is evidently first necessary to devise means of interpreting such a pattern once it is at hand.

Velocity vectors and components. The motion of a fluid, like that of a solid, is described quantitatively in terms of the characteristic known as *velocity*. In dealing with a solid, however, it is generally sufficient to measure the velocity of the body as a whole, whereas the motion of a fluid may be quite different at different points of observation. At any such point, nevertheless, the velocity completely defines the rate of motion at a given instant, in that it is a measure not only of the *speed* at which the fluid is passing that point but also of the *direction* in which the fluid is moving. Velocity is thus a *vector* quantity, for it possesses both magnitude and direction. The vector is usually represented by an arrow, as shown in Fig. 1, the length of the arrow being proportional to the magnitude of the velocity, and the orientation of the arrow indicating the direction of the flow.

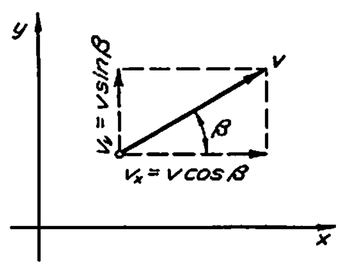


FIG. 1. Velocity vector and rectilinear components.

Owing to its vector properties, a velocity may be resolved into components in any desired directions (such as the rectilinear directions x and y of Fig. 1). Likewise, velocity components at a given point may be combined vectorially to yield their resultant, a procedure which is of particular importance in problems involving *relative motion*. For instance, to an observer in an airplane the air at a point which is fixed in relation to the plane will appear to have a definite direction and speed; the actual air velocity, however, is the vector sum of the velocity of the air relative to the plane and the velocity of the plane itself, as indicated in Fig. 2. Such vector addition is perhaps most readily accomplished by resolving each velocity into rectilinear components, which may then be added algebraically to obtain the components of the resultant.

Stream lines. If one had a series of photographs of smoke leaving a chimney, one might roughly indicate thereon by means of a series of

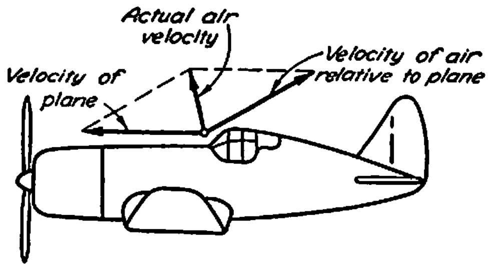


FIG. 2. Vector diagram for relative motion.

arrows the speed and direction of the air currents at a number of typical points. Velocity diagrams of this nature would be complete, of course, only if arrows were drawn at each and every possible point, but then the result would be hopelessly confused. On the other hand, a very satisfactory representation of the flow as a whole at any instant would be obtained by sketching in a series of curves in such manner that the velocity vectors for all points lying upon the curves would meet them tangentially, as shown in Fig. 3. Such curves are known as *stream lines.*

A stream line may thus be defined as a line which shows, through tangency to the velocity vector, the instantaneous direction of flow at every point over its entire length. It follows as a corollary that there can be no flow across a stream line at any point; in other words, the velocity vector necessarily has a zero component at right angles to the stream line to which it is tangent. In general, instantaneous stream lines will converge or diverge as they curve through space, for the velocity usually varies in magnitude and direction from point to point throughout a moving fluid. Once the stream-line pattern is at hand, of course, it is no longer necessary to include the individual velocity vectors, because the *direction* of flow in every region may be

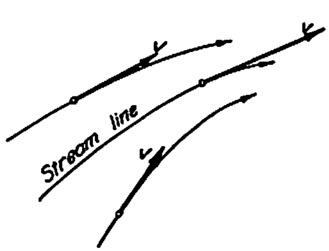
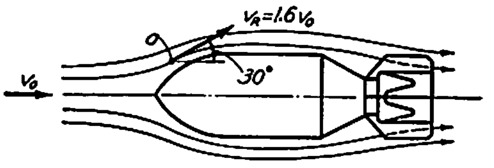


FIG. 3. Stream lines and velocity vectors.

seen from this pattern at a glance; that the stream-line configuration also indicates the *magnitude* of the velocity at all points will be shown in the following section.

Example 1. Tests upon a bomb held stationary in the air stream of a wind tunnel yielded (by means of a series of smoke filaments) the stream lines shown. At point a, where the inclination of the stream line is 30°, the velocity v_R relative



to the bomb was found to be 1.6 times as great as the velocity v_0 of the approaching air. What resultant velocity would this indicate when the bomb was falling at the rate of 500 feet per second? Sketch the corresponding instantaneous stream line.

Since the rate of fall corresponds to the velocity of the air stream in the wind-tunnel tests (i.e., $v_B = v_0$) the relative velocity at point a will be

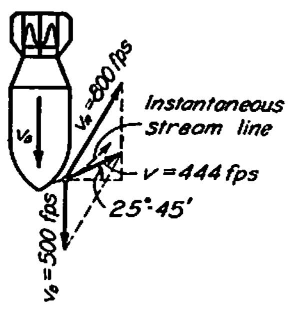
$$v_R = 1.6\, v_B = 1.6 \times 500 = 800 \text{ fps}$$

The resultant velocity is then the vector sum of the relative velocity and the bomb velocity:

$$v = v_R \mathbin{+\!\!\!\!\rightarrow} v_B$$

In terms of vertical and horizontal components,

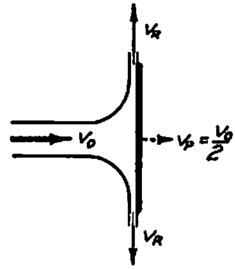
$$v_y = v_R \cos 30° - v_B$$
$$= 800 \times 0.866 - 500 = 193 \text{ fps}$$
$$v_x = v_R \sin 30° = 800 \times 0.5 = 400 \text{ fps}$$
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{400^2 + 193^2} = 444 \text{ fps}$$
$$\tan \beta = \frac{v_y}{v_x} = \frac{193}{400} = 0.482$$
$$\beta = 25° \, 45'$$



The instantaneous stream line would therefore be inclined at the angle β to the horizontal at this point, as indicated in the sketch.

PROBLEMS

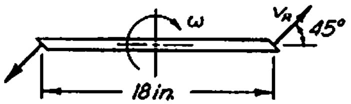
1. When a jet of water is deflected by a plate held at right angles to the jet axis, the speed of the water relative to the plate is the same before and after deflection. If the plate is moved in the direction of the jet at one-half the absolute jet velocity, through what absolute angle will the jet be deflected?



PROB. 1.

2. An airplane is observed to travel due north at a speed of 150 miles per hour in a 50-mile-per-hour wind from the northwest. What is the apparent wind velocity observed by the pilot?

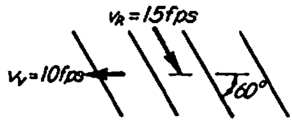
3. Jets at the two ends of the 18-inch rotating arm of a lawn sprinkler direct the water in the plane of rotation at an angle of 45° relative to the arm. If the relative jet velocity is 30 feet per second and the arm makes 150 revolutions per minute, determine the actual velocity of the water as it leaves the sprinkler.



PROB. 3.

4. The air around the front of a baseball necessarily has the same velocity as that part of the ball with which it is in contact. If a ball is thrown horizontally with a speed of 75 feet per second and a back spin of 15 revolutions per second, what will be the air velocity at the foremost point of contact? (Assume a ball diameter of 3 inches.)

5. Water passes through a series of moving vanes inclined at an angle of 60° to their direction of motion. If the water velocity relative to the vanes is 15 feet per second and the vanes move with a speed of 10 feet per second, determine the actual velocity of the water and show in a sketch the inclination of the stream lines.



PROB. 5.

5. EQUATIONS OF CONTINUITY

Continuity of flow through a stream tube. Just as stream lines may be passed through a moving fluid in such manner as to indicate the direction of motion at every point, a tube-like surface bounded by such stream lines may be considered to enclose an elementary portion of the flow. Such an imaginary surface, known as a *stream tube*, is illustrated in Fig. 4. If the cross-sectional area of the tube is sufficiently small, the velocity at its midpoint will closely represent in magnitude and direction the average velocity for the section as a whole. In differential terms, therefore,

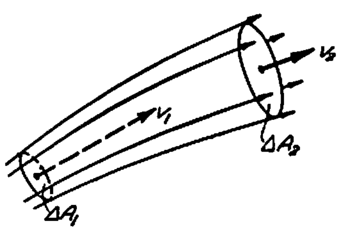


FIG. 4. Velocity variation through a stream tube.

the *volume rate of flow* past any normal cross section of an elementary stream tube will be

$$dQ = v\,dA \tag{1}$$

in which dQ has the dimension of volume per unit time or [length^3/time], v that of distance per unit time or [length/time], and dA that of area or [length^2].

Since the stream tube is bounded by stream lines, it is evident from the definition of the stream line that no fluid whatever can pass through the tube wall. From the law of conservation of mass, moreover, it is clear that fluid matter can be neither created nor destroyed. If, for the present, it is further specified that fluid matter is not expanded or compressed during motion, then at any instant flow must take place past all successive cross sections of a stream tube at the same mass rate. In other words, the mass of fluid passing one cross section per unit of time must equal, simultaneously, the mass per unit time passing every other cross section. But, if the mass per unit volume of the fluid is assumed for the present to remain constant, the volume of fluid passing every section of a given tube per unit time must then also be the same. Therefore, if the mass density of the fluid does not change, the volume rates of flow past all successive cross sections of a stream tube must be equal at any instant. This elementary principle is expressed in the following basic *equation of continuity*:

$$v_1\,dA_1 = v_2\,dA_2 = v_3\,dA_3 \cdots \tag{2}$$

Evidently, the instantaneous velocity of flow must vary *inversely* with the cross-sectional area of the tube.

Before proceeding farther, two circumstances warrant clarification: Equations such as (1) and (2) frequently involve areas and lengths of differential magnitude, whereas the corresponding figures invariably show areas and lengths of small but finite magnitude; this is due primarily to the fact that quantities which are sufficiently minute to make the differential equations exact cannot well be illustrated to scale without losing all semblance of stream-line curvature; on the other hand, the equations are often applied to stream tubes of finite cross-sectional area as a first approximation, the actual error remaining small so long as the curvature of the tube and the velocity variation across the section are not excessive. It may also have been noted that the stream lines and stream tubes appearing in the illustrations show no suggestion of the *turbulence* so often present in fluid motion. Although the concept of stream lines is quite as applicable to flow with turbulence as to flow without, in the former instance the pattern as a whole can become extremely complicated; when turbulence exists, therefore, it is customary to represent by the stream lines only the *average* pattern of motion, a procedure which will be followed throughout this book.

Continuity relationships for two-dimensional flow. In some cases of flow the velocity vector will have components in only two coordinate directions, the component in the third direction being everywhere equal to zero. Under such circumstances the flow is said to be *two-dimensional,* since all the stream lines must then lie in parallel planes. In two-dimensional flow, therefore, all such parallel planes will display the same flow pattern. The intersection of a stream tube with one such plane will be simply two stream lines, as indicated in Fig. 5, and the product of the velocity and the normal distance dn between these lines will correspond to a volume rate of flow *per unit width of section,*

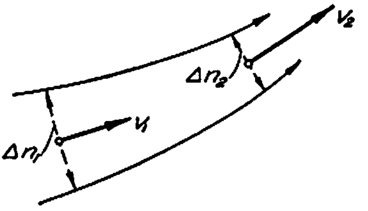


FIG. 5. Velocity variation in two-dimensional flow.

$$dq = v\,dn \tag{3}$$

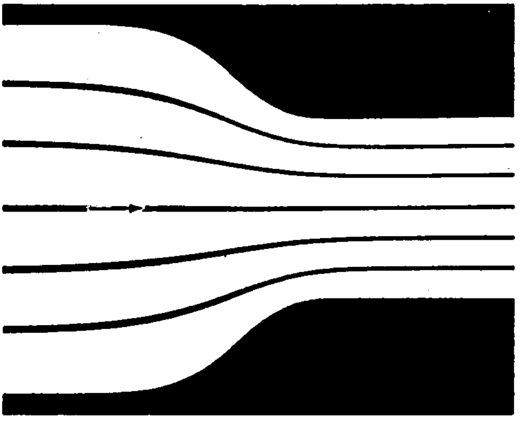


PLATE II. Stream lines at a two-dimensional boundary contraction, made visible by the injection of dye into water flowing between closely spaced glass plates.

dq necessarily having the dimension [volume/time/length] or simply [length^2/time]. The differential equation of continuity for two-dimensional motion thus becomes

$$v_1\,dn_1 = v_2\,dn_2 = v_3\,dn_3 \cdots \tag{4}$$