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Magnetic fluid axisymmetric volume on a horizontal plane near a vertical line conductor in case of non-wetting

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ABSTRACT

Static shapes of a magnetic fluid axisymmetric volume on a horizontal plane in the magnetic field of a vertical line conductor are studied theoretically in case of non-wetting while the current is slowly changing in a quasi-static manner. The possibility of the fluid shape hysteresis for a cyclic increase and decrease of the current and of spasmodic changes at certain values of the current is investigated.

1. Introduction

The behavior of the free surface of an infinite magnetic fluid (MF) volume near a vertical line conductor with current is considered to be a classical problem in ferrohydrodynamics and it was firstly developed in [1]. In [2] a rapid jump of the MF ascension height was discovered theoretically and confirmed in the experiment for some value of the current in case of small magnetic fields. In [1,2] only the case of wetting was considered. The problem of a finite MF volume on a horizontal plane near a vertical line conductor was studied in [3] in case of wetting for any magnetic fields. It was shown that the fluid shape hysteresis for a cyclic increase and decrease of the current and spasmodic changes at certain values of the current are possible. In this paper we consider a similar problem as in [3] but in case of non-wetting. Due to that, instead of a pyramidal shape in case of wetting (where the base is the widest), the diameter of such drop in case of non-wetting is maximal at some intermediate value of its height.

2. Theory

We consider a heavy, incompressible, homogenous, isothermal MF drop of the finite volume V on a horizontal plane in the magnetic field of a vertical line conductor of the radius r_0 with the current I (Fig. 1). The case of non-wetting is considered, that is to say, the MF does not wet solid boundaries, so $90° < \theta_1, \theta_2 \le 180°$, where θ_1 is the wetting angle of the line conductor, θ_2 is the wetting angle of the horizontal plane. The MF is immersed in a non-magnetic liquid with the same density (the case of hydroimponderability). The MF has a free axially symmetric surface of revolution $z = h(r)$, $r = \sqrt{x^2 + y^2}$ (the axis z is directed along the axis of the line conductor, see Fig. 1).

In this geometry, the magnetic field of the line conductor $|\mathbf{H}|$ is not deformed by the MF and $|\mathbf{H}| = H$, $H(r) = 2I/(cr)$, where c is the speed of light in vacuum [4]. We consider that the MF magnetization M can be described by the Langevin law [5]: $M(\xi) = M_S L(\xi)$, $L(\xi) = \coth\xi - 1/\xi$, $\xi = mH/(kT)$, $m = M_S/n$. Here M_S is the MF saturation magnetization, m is the magnetic moment of one ferromagnetic particle, n is the number of ferromagnetic particles per unit volume, T is the fluid temperature, k is the Boltzmann constant, ξ is the Langevin parameter which corresponds to the current in a line conductor.

We use the hydrostatic equation [1]:

$$-\nabla p_j + M_j \nabla H_j + \rho_j \mathbf{g} = \mathbf{0}, \quad j = f, l. \tag{1}$$

Here the indices f and l denote the MF and the non-magnetic ($M_l = 0$) liquid surrounding the MF, p is the fluid pressure, ρ is the fluid density, g is the gravitational acceleration. We also use the boundary condition on the free surface $h(r)$ [1]:

$$p_l - p_f = \pm 2\sigma K. \tag{2}$$

Here σ is the interfacial tension between the MF and the non-magnetic liquid, $K = K(h', h'')$ is the mean curvature of the surface. The signs "+"("−") should be chosen when the non-magnetic liquid is situated above (beneath) the MF.

From Eqs. (1) and (2) we derive a general inhomogeneous non-linear second-order differential equation and get a general analytical solution for any axially symmetric shape of the MF free surface $h(r)$ in any axisymmetric magnetic field in the non-dimensional form [3]. Here we need to describe separately the non-dimensional upper contact surface of the fluids $h_1^*(r^*) \ge h^*(r_d^*)$ ($r_d^* = r_d/r_0$ is the non-dimensional

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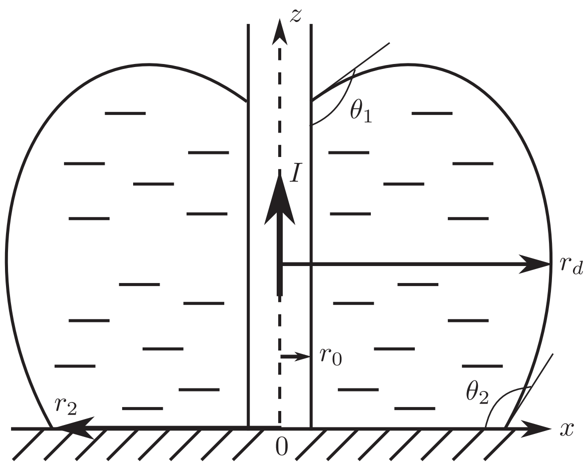

Fig. 1. Schematic image of the MF volume.

distance, or the MF thickness, between the center of a line conductor and a point, where the tangent line to the free surface of the MF is vertical) and the non-dimensional lower contact surface of the fluids $h_2^*(r^*) \le h^*(r_d^*)$:

$$h_i^*(r^*) = -\int_{r^*}^{r_d^*} \frac{G_i}{\sqrt{1-G_i^2}} dr^* + D_i, \quad i = 1, 2, \tag{3}$$

$$G_i(r^*) = \frac{C_i}{r^*} + B_i r^* + (-1)^i \frac{P_1(r^*, \xi_0)}{r^*} \int_{r_i^*}^{r^*} r^* P^*(r^*, \xi_0) dr^*, \tag{4}$$

$$P^*(r^*, \xi_0) = \ln \frac{\sinh(\xi_0 H^*)}{\xi_0 H^*}, \; B_i = \frac{(-1)^i r_0 (p_{0f} - p_{0l})}{2\sigma}. \tag{5}$$

Here $r_i^* = r_i/r_0$ is the non-dimensional radius of points, where the surface $h_i^*(r^*)$ touches the line conductor or the horizontal plane, correspondingly, so $r_1^* = 1$. Here B_i, C_i, D_i are unknown constants, p_{0f}, p_{0l} are constants of integration of Eq. (1), and some other non-dimensional parameters are introduced: $r^* = r/r_0$, $H^* = 1/r^*$, $\xi_0 = 2mI/(cr_0kT)$, $P_1 = nkTr_0/\sigma$. Later, the signs "*" are omitted and parameters are considered as non-dimensional, unless otherwise specifically agreed.

On contact lines of three media, for $r = 1$ and $r = r_2$, the Jung condition should be satisfied. It gives the following boundary conditions:

$$G_1(r = 1) = -\cos\theta_1, \; G_2(r = r_2) = \sin\theta_2. \tag{6}$$

For $r = r_d$ the function $h_1(r)$ ($h_2(r)$) is decreasing (increasing) and at this point the tangent line to the free surface of the MF is vertical, so $G_1(r = r_d) = -1$ and $G_2(r = r_d) = 1$. From these equations and Eqs. (6) we get the dependence of the constants B_1, C_1 on the radius r_d and of the constants B_2, C_2 on the radii r_d and r_2:

$$B_1 = \frac{P_1 \int_1^{r_d} rP(r, \xi_0)dr + \cos\theta_1 - r_d}{r_d^2 - 1}, \; C_1 = -\cos\theta_1 - B_1,$$

$$B_2 = \frac{-P_1 \int_{r_2}^{r_d} rP(r, \xi_0)dr - r_2\sin\theta_2 + r_d}{r_d^2 - r_2^2}, \; C_2 = r_2\sin\theta_2 - B_2 r_2^2. \tag{7}$$

It should be noted that Eqs. (5) give the following relation between the constants B_1 and B_2:

$$B_2(r_2, r_d) = -B_1(r_d). \tag{8}$$

The constant $D = D_1 = h_1(r_d) = D_2 = h_2(r_d)$ could be determined from Eq. (3) and $h_2(r_2) = 0$, as following:

$$D = \int_{r_2}^{r_d} \frac{G_2}{\sqrt{1-G_2^2}} dr. \tag{9}$$

The MF volume V (Fig. 1) could be calculated by the following formula:

$$V = 2\pi \int_1^{r_d} rh_1 dr - 2\pi \int_{r_2}^{r_d} rh_2 dr. \tag{10}$$

If we set the values of the problem parameters and fix the values of the variables ξ_0 and r_d, the system of Eqs. (3)–(9) allows us to determine the MF shape $h(r)$ in a state of hydrostatic equilibrium, if it exists, or to show that for these fixed values of parameters and variables there is no static MF shape.

3. Numeric computation

A program is written in the computer simulation software Maple to determine the MF shape and its volume. To fulfill the assumption of non-wetting with respect to all solid interfaces, i.e., the plane and the line conductor, different combinations "neutral liquid – MF – solid surface material" could be used. For example, in [6] the solid surface was made of polymethyl methacrylate (PMMA) chemically modified by rinsing it with a commercial antispread liquid (TE1403DA53, Dr. Tillwich GmbH, Germany); the MF was a water-based ferrofluid (EMG705, Ferrotec Corp., USA); the neutral liquid was a mixture of hydrogenated terphenyl and 1-bromonaphthalene (No. 18095, Cargille Labs, USA), which is immiscible with the MF. Furthermore, in [7] the solid surface was also made of PMMA; the MF was also a water-based ferrofluid; the neutral liquid was a transformer oil. We use the following dimensional parameters (similar to the examples mentioned above) for the numeric calculations: θ_1 , $\theta_2 = 120°$, $T = 300$ °K, $M_s = 80$ G , $n = 1.1 \cdot 10^{17}$ cm^{-3}, $\sigma = 7$ dyn/cm and $r_0 = 0.065$ cm.

The dependence $V = V(r_d)$ for different fixed values of ξ_0 is presented in Fig. 2. It is shown that there are four critical values of ξ_0: $\xi_{01} = 0.5875$, $\xi_{02} = 0.6235$, $\xi_{03} = 0.6245$, $\xi_{04} = 0.6738$, for which the form of the dependence $V(r_d, \xi_0)$ strongly changes. If $0 \le \xi_0 \le \xi_{01}$, the dependence $V = V(r_d, \xi_0)$ is a monotonous one (lines 1, 2 in Fig. 2). If $\xi_0 > \xi_{01}$, the dependence is multivalued (only in case of non-wetting) as it is shown in Fig. 3: for one value of r_d there are up to three different volumes V (line 3), because Eq. (8) can have up to three different roots r_2. The middle solutions presented by the dashed line in Figs. 2 and 3 are usually unstable. If $\xi_0 = \xi_{02}$, the line $V = V(r_d, \xi_{02})$ has an inflection point. If $\xi_{02} < \xi_0 < \xi_{03}$, the dependence has a maximum and a minimum: one value of V corresponds to up to three values of r_d (line 4). If $\xi_{03} \le \xi_0 < \xi_{04}$, the dependence has two branches and each branch tends to its own asymptote (line 5). If $\xi_0 \ge \xi_{04}$, the dependence $V = V(r_d, \xi_0)$ is

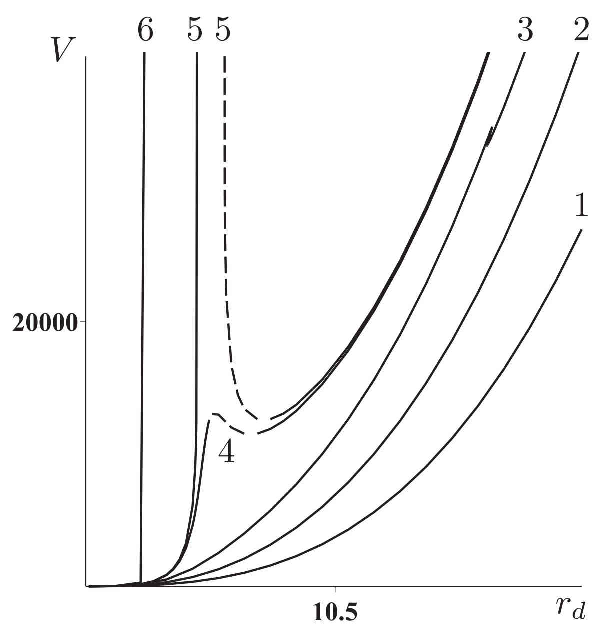

Fig. 2. The dependence $V = V(r_d)$ for different values of ξ_0: 1) $\xi_0 = 0$, 2) $\xi_0 = 0.5$, 3) $\xi_0 = 0.59$, 4) $\xi_0 = 0.624$, 5) $\xi_0 = 0.625$, 6) $\xi_0 = \xi_{04} = 0.6738$.

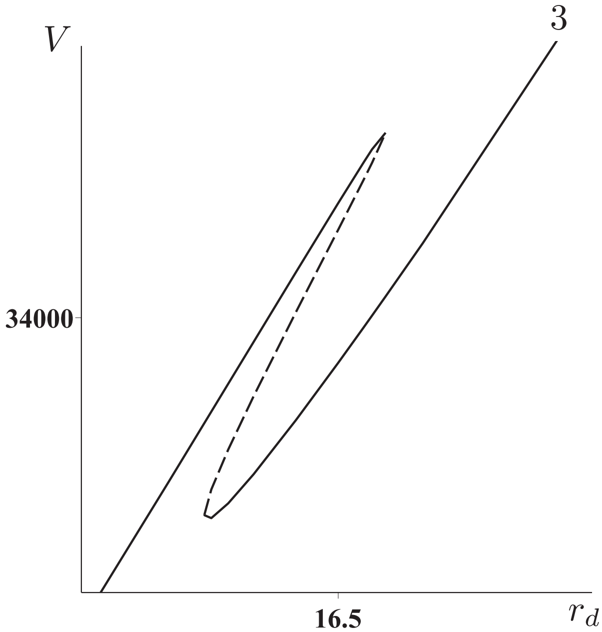

Fig. 3. Details of the dependence $V = V(r_d)$ for $\xi_0 = 0.59$ (line 3 in Fig. 2).

a monotonous one and it tends to its asymptote (line 6).

Thus, for both cases of wetting [3] and non-wetting under the same geometry of the interfaces, the MF shape is liable to bistability under the change of the current. It means that spasmodic changes of r_d and the current-induced shape hysteresis should occur for the big enough drops $V > V_{c1}$ ($V > V_{c2}$) and for $\xi_0 > \xi_{01}$ ($\xi_0 > \xi_{02}$): the MF shape changing while the current is increasing does not coincide with the MF shape changing while the current is decreasing. The critical value of volume $V_{c1} = 30347$ (8.3 cm^3) is determined by the inflection point for $\xi_0 = \xi_{01} = 0.5875$ (10.9 A); $V_{c2} = 10852$ (3 cm^3) is determined by the inflection point for $\xi_0 = \xi_{02} = 0.6235$ (11.5 A).

4. Conclusion

The method to calculate the magnetic fluid shape in a state of hydrostatic equilibrium on the horizontal plane at the constant value of the current in the vertical line conductor is proposed in case of non-wetting. For some fixed values of the current in the conductor and of the magnetic fluid volume, various static shapes are obtained numerically. Like in case of wetting, the bistability could also lead to the presence of spasmodic and hysteresis phenomena in case of non-wetting.

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