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On Strong Shock Waves of Annihilation

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Abstract—In the framework of special relativity theory, we give a piecewise-constant solution to the one-dimensional problem of collision of a combination of particles and antiparticles with a flat wall which results in an annihilation shock wave. We assume that there is a complete transition of antimatter into the energy of remaining matter and outgoing radiation directed from the wall and, in its turn, separated from the incoming flow by a null rupture surface. The initial pressure in the mixture is neglected. We have constructed and investigated the Taub annihilation adiabat, including the Jouguet mode. Several limiting cases are considered as well, including a small concentration of antimatter, a small difference between the concentrations of matter and antimatter, the absence of directional radiation, and nonrelativistic velocity of the incoming flow.

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1. INTRODUCTION

In the framework of general relativity, in particular, in Newtonian mechanics, Refs. [1–4] show that a gravitational collapse of any mass of gas with a parabolic compression rate, for example, the Universe, can turn to hyperbolic dispersion if a part of the rest mass of matter transforms to thermal energy in a detonation wave. At the same time, all physical variables are limited. This paper is devoted to a special case of this problem which takes into account the annihilation nature of such a process, accompanied by possible directional radiation.

Sufficiently energetic nucleons consist of three valence quarks bound by gluons, and according to the Heisenberg relation, a "sea" of pairs of quarks and antiquarks. The energy and momentum of the nucleons are distributed among all these particles [5]. Antiquarks in the collision of nucleons can combine in free antinucleons. As a result of annihilation, nucleons and antinucleons disintegrate into smaller particles: pions, muons, and so on, up to neutrinos and photons [6]. We assume that the annihilation reaction occurs in a thin shock layer (the detonation model of Neumann, Zel'dovich, and Doering [7]). Estimates of the thickness of this layer in the case of a nuclear detonation, considering energy losses due to formation of electron-positron pairs, are given in [8]. There can be found a review of the earlier literature on relativistic shock waves. In this paper, we consider the problem of the process of energy release in the mixture of initially separated particles and antiparticles at their annihilation, taking into account the outgoing radiation.

In the framework of the special relativity theory, we give a piecewise constant solution to the one-dimensional problem of a collision of a combination of particles and antiparticles with a flat wall, simulating the presence of a center of symmetry. As a result, two discontinuity surfaces are formed, which split the gas flow into three areas: the area of motion of the initial mixture without radiation—0, the area of motion of the same mixture with the outgoing counter freely penetrating radiation—1 and the quiescence area of the remaining matter under high pressure near the wall—2. The surface between of the first two areas moves at the speed of light. The second surface is a shock wave, in which annihilation of matter (m) and antimatter (a) occurs.

To exclude the factor of the electromagnetic field, we assume that annihilation involves uncharged particles such as neutrons and antineutrons [9]. The nature of radiation—in the form of photons or neutrinos—is considered to be insignificant. The pressure in the initial mixture is equal to zero, i.e., the shock wave of annihilation is strong [10].

We have constructed and investigated the Taub annihilation adiabat [11] presented in a parametric form. In the absence of radiation, we have considered the Jouget mode. Along with the construction of solutions of the full problem, a number of limiting cases are investigated: a small concentration of antimatter; a small difference between the concentrations of matter and antimatter; the absence of directional radiation; a non-relativistic velocity of the incoming flow.

2. THE TAUB ADIABAT

We select such a system of units where the speed of light $c = 1$ and the total density of the initial mixture $\rho_m + \rho_a = 1$. Hereinafter, we consider only the tensor components with indices 0 and 1 and use the Minkowski metric $ds^2 = dt^2 - dx^2$.

The motion of the gas satisfies the following equations:

In area 0:

$$\nabla_j(\rho_m u_1^i u_1^j) = 0, \quad \nabla_j(\rho_a u_1^i u_1^j) = 0, \tag{1}$$

where $u_{i1}u_1^i = 1$, $(u_{i1}) = (1,\ -v)/\sqrt{1-v^2}$ is the initial 4-velocity, $-1 < v < 0$.

The total energy-momentum tensor is

$$T_0^{ij} = \rho_m u_1^i u_1^j + \rho_a u_1^i u_1^j = u_1^i u_1^j.$$

We assume that the velocities of matter and antimatter are the same.

In area 1:

Equations (1) and

$$\nabla_j(q^i q^j) = 0, \tag{2}$$

where $q_i q^i = 0$, $(q_i) = (q,\ -q)$ is a null vector of directional radiation. The energy-momentum tensor is $T_1^{ij} = T_0^{ij} + q^i q^j$.

In area 2:

$$\nabla_j T^{ij} = 0, \quad T^{ij} = (p+\varepsilon)u^i u^j - p\eta^{ij}, \tag{3}$$

where p is pressure, $\varepsilon = \rho + p/(\gamma - 1)$ is the internal energy density of a perfect gas, $(u_i) = (1,\ \ 0)$ is the 4-velocity, ρ is the density of the remaining matter beyond the shock wave, $\nabla_i(\rho u^i) = 0$, and (η^{ij}) is the Minkowski tensor.

Due to piecewise constancy of the solutions, all equations (1)–(3) are satisfied identically. Let us consider the conditions on discontinuities. On the surface between the areas 0 and 1, moving at the speed of light, the density and velocity of matter and antimatter are considered to be continuous. Conserving the energy-momentum flow $[T^{ij}n_j]_0^1 = 0$, since the 4-normal and radiance vectors are proportional, $n_i \sim q_i$, $\quad n_i n^i = 0$, is also performed identically.

On the shock wave between areas 1 and 2,

$$[T^{ij}n_j]_1^2 = 0, \quad [\rho u_n]_1^2 = 0 \tag{4}$$

express the conservation laws of energy, momentum, and the mass which has not annihilated, where u_n is the projection of the 4-velocity on the 4-normal $(n_i) = (D,\ -1)/\sqrt{1-D^2}$ to the surface, and D is the speed of the shock wave, $0 < D < 1$.

We suppose that the annihilated mass turns to directed radiation according to the ratio

$$q_n q_u = -2\rho_a u_{n_1} k,$$

where q_n and q_u are the projection of the directed radiation vector on the normal and 4-velocity vectors, and k is the conversion factor of annihilated mass into radiation ($0 \le k < 1$). Then,

$$q^2 = \frac{2\rho_a(D-v)k}{(1-D)(1-v)}.$$

Let α be the concentration of matter, $1/2 < \alpha \le 1$. We derive from Eqs. (4):

$$\rho = \frac{(2\alpha-1)(1-v/D)}{\sqrt{1-v^2}},$$

$$\varepsilon D = -\frac{2k(1-\alpha)(D-v)}{1-v} + \frac{D-v}{1-v^2},$$

$$p = \frac{2k(1-\alpha)(D-v)}{1-v} - \frac{v(D-v)}{1-v^2}. \tag{5}$$

Let us introduce the specific internal energy

$$U = \frac{\varepsilon}{\rho} = -2k\frac{1-\alpha}{2\alpha-1}\sqrt{\frac{1+v}{1-v}} + \frac{1}{(2\alpha-1)\sqrt{1-v^2}}.$$

Then

$$(U-1)(\gamma-1) = \frac{p}{\rho}$$
$$= \left(\frac{2k(1-\alpha)}{2\alpha-1}\sqrt{\frac{1+v}{1-v}} - \frac{v}{(2\alpha-1)\sqrt{1-v^2}}\right).$$

The last equation and (5) yield the velocity of the annihilation wave

$$\frac{D}{\gamma-1}$$
$$= \frac{-2k(1-\alpha)(1+v) + 1 - \sqrt{1-v^2}(2\alpha-1)}{2k(1-\alpha)(1+v) - v},$$

note that the the denominator never becomes zero.

Finally, for the thermodynamic parameters we have

$$p = \frac{1}{1-v^2}\Big((\gamma-1)[-2k(1-\alpha)(1+v) + 1$$
$$- (2\alpha-1)\sqrt{1-v^2}] - 2k(1-\alpha)(1+v)v + v^2\Big),$$
$$U = \frac{-2k(1-\alpha)(1+v)+1}{(2\alpha-1)\sqrt{1-v^2}}.$$

Thus we have the Taub annihilation adiabat in the parametric form (the parameter $-1 < v < 0$), which connects the pressure and specific internal energy in the area beyond the discontinuity. In the classical case, as is known, the specific volume $1/\rho$ is used instead of U is the Hogoniot adiabat in Newtonian gas dynamics [7].

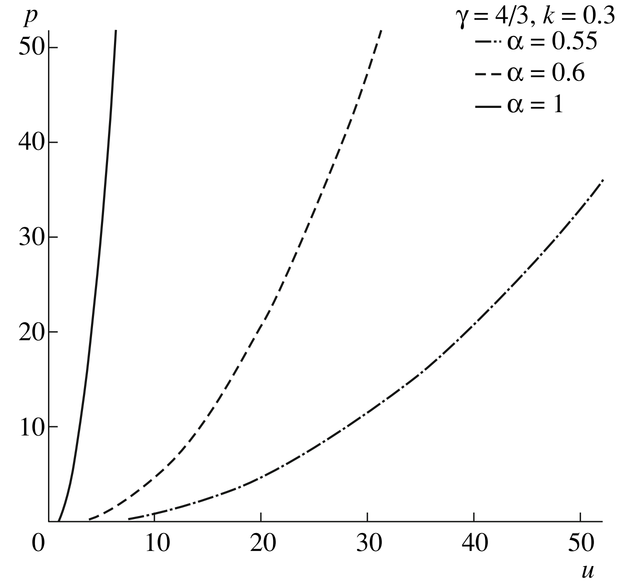

Fig. 1.

If we fix the adiabatic index $1 < \gamma \leq 2$ and the coefficient k, then the the smaller is α, the greater amount of specific internal energy corresponds to a predetermined pressure (Fig. 1).

If we fix γ and α, then the greater amount of the annihilated rest mass is transformed into directed radiation, the smaller amount of specific internal energy corresponds to predetermined pressure (Fig. 2).

And if we fix α and k, then the smaller is gamma, the greater amount of specific internal energy corresponds to a predetermined pressure (Fig. 3).

3. SPECIAL CASES

Let us consider some limiting cases. We discard the small parameters of higher orders in the expansion.

I. Let $k = 0$ (the absense of directional radiation). Then

$$\rho = \frac{(2\alpha - 1)(1 - v/D)}{\sqrt{1 - v^2}},$$

$$\frac{p}{\gamma - 1} + \rho = \frac{1 - v/D}{1 - v^2},$$

$$p = -\frac{(1 - v/D)vD}{1 - v^2},$$

whence

$$D = \frac{\gamma - 1}{v}\left((2\alpha - 1)\sqrt{1 - v^2} - 1\right),$$

and $D \leq a$, where a is the velocity of sound in area 2:

$$a = \sqrt{\frac{\partial p}{\partial \varepsilon}\Big|_\rho} = \sqrt{\frac{\partial p}{\partial \rho} \cdot \left(\frac{\partial \varepsilon}{\partial \rho}\right)^{-1}} = \sqrt{\frac{(\gamma - 1)\gamma p}{(\gamma - 1)\rho + \gamma p}}.$$

The equality $D = a$ corresponds to the Jouget mode [7].

We perform a hyperbolic substitution of variables: $v = \tanh\theta, 1/\sqrt{1 - v^2} = \cosh\theta, v/\sqrt{1 - v^2} = \sinh\theta$. Then:

$$D = \frac{(\gamma - 1)(2\alpha - 1 - \cosh\theta)}{\sinh\theta},$$

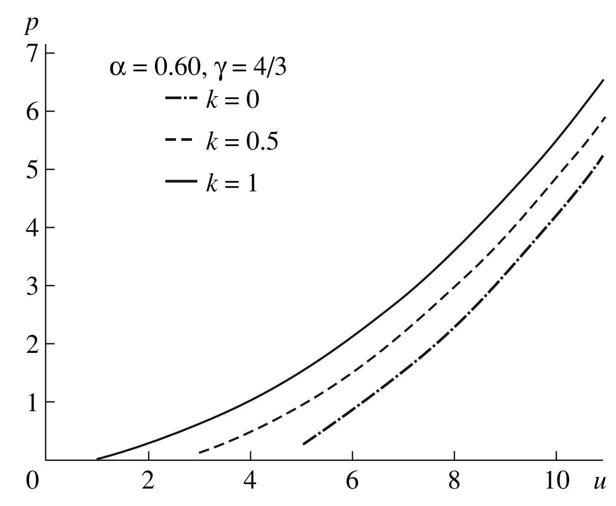

Fig. 2.

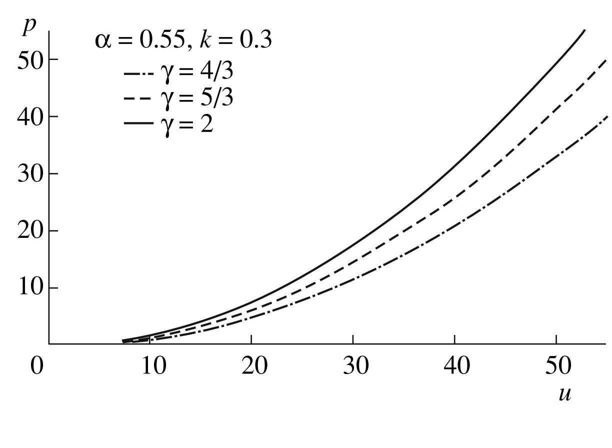

Fig. 3.

$$\rho = \frac{(2\alpha-1)(-\gamma\cosh^2\theta + (\gamma-1)(2\alpha-1)\cosh\theta + 1)}{(2\alpha-1-\cosh\theta)(\gamma-1)},$$

$$p = \gamma\cosh^2\theta - (\gamma-1)(2\alpha-1)\cosh\theta - 1,$$

whence

$$\cosh\theta = (2\alpha-1)\left(\frac{p}{(\gamma-1)\rho}+1\right) = (2\alpha-1)U > 1.$$

The result is the Taub annihilation adiabat in an explicit form:

$$p = (2\alpha-1)^2U(\gamma U + 1 - \gamma) - 1.$$

At the point $p = 0$,

$$U = \frac{\gamma-1}{2\gamma} + \sqrt{\left(\frac{\gamma-1}{2\gamma}\right)^2 + \frac{1}{\gamma(2\alpha-1)^2}}.$$

Let us consider additional restrictions:

Ia. Let $|v| \ll 1$ (Newtonian limit) and $\alpha \to 1$ (a small concentration of antimatter), $1-\alpha = \omega \to 0$, and let $\omega \sim v^2$.

Then,

$$p \approx U(\gamma U + 1 - \gamma) - 1$$

and

$$\cosh\theta \approx 1 + v^2/2 = (1-2\omega)U = (1-2\omega)(1+c_VT),$$

where T is the absolute temperature, and c_V is the specific heat capacity at constant volume. So, $v^2/2 + 2\omega \approx c_VT$, which is performed at small T.

And from

$$D \approx \frac{\gamma-1}{v}(-2\omega - \frac{v^2}{2}) \le a$$

we have

$$v^2 \ge \frac{4\omega(\gamma-1)}{\gamma+1},$$

where the equality corresponds to the Jouget mode.

In this case, on the (p, V) plane, where $V = 1/\rho$, we have

$$p \approx \frac{4\omega(\gamma-1)}{(\gamma+1)V - (\gamma-1)},$$

2ω is equivalent to specific energy of detonation Q [11], see Fig. 4.

If we decrease ω, the plot comes closer to the axis and to the asymptote, and if $\omega = 0$, then $V = (\gamma-1)/(\gamma+1)$.

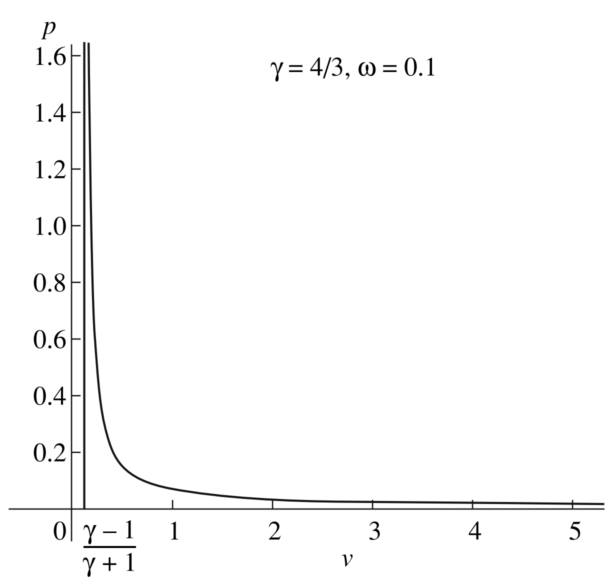

Fig. 4.

The vertical asymptote here is the Hugoniot adiabat for the Newtonian mechanics problem without counterpressure.

Ib. Let us consider the ultrarelativistic case. From $p \gg \rho$ it follows

$$-\frac{vD}{(1-2\omega)\sqrt{1-v^2}} \gg 1,$$

whence either (1) $v^2 \to 1$ and any ω, or (2) $\omega \to 1/2$ and any v.

Ib1.

$$U \approx \frac{1}{\sqrt{1-v^2}(1-2\omega)}, \quad p \approx \frac{\gamma}{1-v^2},$$

the Taub adiabat is

$$p \approx \gamma(1-2\omega)^2U^2.$$

Ib2.

$$U \approx \frac{1}{\sqrt{1-v^2}(1-2\omega)}, \quad p \approx \frac{\gamma-1+v^2}{1-v^2},$$

and the Taub adiabat is

$$p \approx \gamma(1-2\omega)^2U^2 - 1,$$

so that $p \sim 1$.

II. Let now $k \neq 0$.

IIa. Let again $|v| \ll 1$, $\omega \to 0$, and $\omega \sim v^2$.

Then

$$U \approx 1 + \frac{v^2}{2} + 2\omega(1-k),$$

$$p \approx \frac{\gamma+1}{2}v^2 + 2\omega(\gamma-1)(1-k),$$

the Taub adiabat is

$$p \approx (\gamma+1)(U-1) - 4\omega(1-k),$$

or on the (p, V) plane

$$p \approx \frac{4\omega(\gamma-1)(1-k)}{(\gamma+1)V-(\gamma-1)}.$$

If $k=0$, we have the Ia case. And increasing k causes that the plot shown for case Ia goes closer to the axis and to the asymptote, and finally, if $k=1$, $V=(\gamma-1)/(\gamma+1)$.

If $\omega = 0$ [3], which corresponds to a complete absence of antimatter, then

$$p = \frac{1}{1-v^2}\Big((\gamma-1)[1-\sqrt{1-v^2}] + v^2\Big),$$

$$U = \frac{1}{\sqrt{1-v^2}}.$$

Or, excluding v:

$$p = \gamma U^2 - (\gamma-1)U - 1.$$

IIb. Finally, let us consider the ultrarelativistic case for $k \neq 0$: $p \gg \rho$ gives us

$$\frac{1-2k\omega(1+v)}{(1-2\omega)\sqrt{1-v^2}} \gg 1$$

whence (1) $v^2 \to 1$ and any ω, or (2) $\omega \to 1/2$ and any v.

IIb1. Let $1+v \equiv \Delta v \to 0$, then $v^2 \approx 1-2\Delta v$,

$$U \approx \frac{1}{(1-2\omega)\sqrt{2\Delta v}},$$

$$p \approx \frac{1}{2\Delta v}\big(\gamma - (\gamma-1)(1-2\omega)\sqrt{2\Delta v}\big),$$

$$p \approx \gamma(1-2\omega)^2U^2.$$

IIb2. Let $1-2\omega = \sigma \to 0$. We have

$$U \approx \frac{1-k(1+v)}{\sigma\sqrt{1-v^2}},$$

$$p \approx \frac{1}{1-v^2}[(\gamma-1) + v^2 - k(1+v)\frac{\gamma+1}{2}].$$

IIb3. If $v^2 \to 1$ and $\omega \to 1/2$ together and $v \sim \omega$,

$$p \approx (\gamma-1)[1-\sigma-\frac{k}{2}(1+v-\sigma)] - kv,$$

$$U \approx \frac{1-k(1+v-\sigma)}{\sigma}$$

and

$$p \approx (\gamma-1)\frac{1+U\sigma}{2} - 1 + k + U\sigma.$$

4. CONCLUSION

This problem allows us to understand the basic laws of propagation of a shock wave of annihilation.

Annihilation of matter, as the loss of mass, helps us to explain the possibility of stopping the collapse of dust. Also, the solution indirectly points at possible explanations of the observed expansion of the Universe due to partial annihilation of matter during precollapse.

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REFERENCES

1. A. N. Golubiatnikov, *"On spherically symmetric relativistic motion of a gravitating gas in the presence of a strong shock wave,"* DAN USSR **233** (3), 318–321 (1977).
2. A. N. Golubiatnikov, *"A strong explosion and restructuring of equilibrium at relativistic gravitational collapse,"* DAN USSR **251** (6), 1330–1334 (1980).
3. A. N. Golubiatnikov, *"On the mechanism of separation of energy-momentum from the rest mass,"* In *Mechanics. Modern problems* (MSU press, Moscow, 1987), p. 152–157; Reissue: Aeromechanics and Gas Dynamics 1, 73–77 (2002).
4. A. N. Golubiatnikov, *"To form a homogeneous dispersion of gravitating gas,"* Izvestiya RAN. Fluid Mechanics 6, 158–163 (1997).
5. L. B. Okun, *Leptons and Quarks* (Moscow, Nauka, 1989).

6. H. V. Klapdor-Kleingrothaus and A. Staudt, *Teilchenphysik onhe Beschleuniger* (B.G. Teubner Stuttgart, 1995).
7. Ya. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena* (Academic Press, New York, 1967).
8. R. A. Gross, *"Physics of the strong shock waves in gases,"* in *Physics of High Energy Density*, in the Russian edition (Academic Press, New York and London, 1971), p. 273–310.
9. E. Klempt, C. Batty, and J. Richard, *"The antinucleon-nucleon interaction at low energy: annihilation dynamics"* (e-edition, 2008).
10. L. I. Sedov, *Similarity and Dimensional Methods in Mechanics* (CRC Press, 1993).
11. L. D. Landau and E. M. Lifshitz, *Theoretical Physics, Fluid Mechanics* (Butterworth-Heinemann, 1987).